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The order of stress singularity at the vertex in three-dimensional bonded joints

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Abstract

Dissimilar materials are frequently used in industrial products, such as electronic devices. Many investigations on two-dimensional joints so far have been carried out theoretically and experimentally. In this paper, the order of stress singularity at the vertex in three-dimensional bonded joints is investigated. The contour map of the order of stress singularity in a form of power-law singularity on the Dundurs' composite plane in plane strain condition is presented. It is shown that the order of stress singularities at the vertex in three-dimensional bonded joints is different from that in two-dimensional bonded joints with various combinations of material properties.

***Keywords**:* Order of stress singularity, Dundurs' parameters, Bonded joints.

1. Introduction

Consider a bimaterial that consists of two dissimilar homogeneous isotropic material bonded together along their interface. There are two elastic constants each in the two materials, resulting in a total four elastic constants. Dundurs [1-3] has proved that the solution of the stress depends on two composite elastic constants (known as Dundurs constants). Many investigations have been conducted so far concerning joints fabricated from materials with different properties, in order to effectively utilize the feature of each material. Joint structures of bonded metals and ceramics have been used widely in electric devices and mechanical parts. It is known from previous studies that failure occurs and the reliability of the materials decreases due to the occurrence of stress singularity at the cross-point of the free surface and the bonded plane (Fig.1). Fracture and delamination occur often around the vertex of joints. Such problems cause the decrease of reliability of joints.

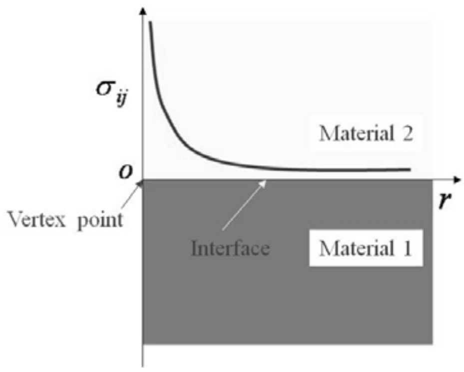

Fig. 1 Stress singularity at vertex point.

Therefore, many studies on the reduction of stress singularity have been carried out theoretically and experimentally [4]. Almost all these studies are focused on two-dimensional stress singularity [5-8]. In a practical point of view of fracture mechanics and application of joints, an analysis of three-dimensional singularities would be useful.

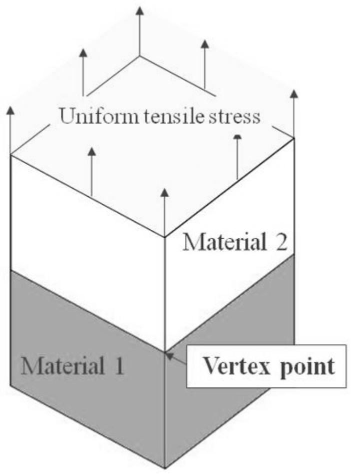

Fig. 2 Vertex point in three-dimensional bonded joints.

There are several investigations on the stress singularity field in three-dimensional elastic materials. Bazant [19] first developed a general numerical procedure for determining three-dimensional stress singularities. Benthem [11-12] examined the singularity exponent of the stress field at the corner point of the free surface with a crack front in a three-dimensional crack. Ghahremani et al. [13] analyzed the elastic anisotropy-induced stress concentrations at triple junctions in three dimensions, and showed that the concentration effect in three-dimensions to be stronger than those obtained for plane strain configurations. When we estimate the strength and reliability of joints, we have to know the order of stress singularity and stress distribution. Then stress intensity factors at the vertex of the joint can be calculated using interpolation method. Furthermore, when the order of stress singularity is arranged on Dundur's parameters plane, it is very useful for various combinations of materials. In this present study, the order of stress singularity at the vertex in three-dimensional bonded joints is investigated and then plotted on Dundur's parameter plane. Graphed results on the planes of Dundur's parameters for a two-dimensional stress state and those for a three-dimensional are compared.

2. Method and Model of analysis

Dundurs introduced the well-known parameters, $\alpha-\beta$, utilizing a description of a stress state in dissimilar materials. The parameters can be expressed using pairs of material properties (G,ν)

$$\alpha=\frac{m_2-m_1\Gamma}{m_2+m_1\Gamma} \tag{1}$$

$$\beta=\frac{(m_2-2)-(m_1-2)\Gamma}{m_2+m_1\Gamma} \tag{2}$$

where

$$\Gamma=\frac{G_2}{G_1} \tag{3}$$

$$m_i=\begin{cases} 4(1-\nu_i) & \text{for plane strain} \\ \dfrac{4}{1+\nu_i} & \text{for plane stress} \quad \text{i=1,2} \end{cases} \tag{4}$$

in which G is the shear modulus and ν is Poisson's ratio. The subscripts of these material properties represent the region of materials.

Since the mechanical properties for all materials are in the range of $G_1, G_2 \geq 0$, $0 \leq \nu_1, \nu_2 \leq 0.5$,

the existence domain of $\alpha - \beta$ is within the boundary enclosed by four straight lines as follows

$$\alpha = \pm 1 \tag{5}$$

$$\beta = \begin{cases} \dfrac{\alpha \pm 1}{4} & \text{for plane strain} \\ \dfrac{3\alpha \pm 1}{8} & \text{for plane stress} \end{cases} \tag{6}$$

FEM formulation using an interpolation function of displacements, considering the stress singularity presented by Yamada and Okumura [17] and Pageau and Biggers [14] is used to analyze the order of stress singularity. We can obtain multiple real as eigen values for the eigen equation of the displacement vector, and examine the order of stress singularity. In previous studies, we found that number of integration points is 20 and the mesh size should be less than 10°x10° that the convergence rating and the time consumption for calculation are optimum as shown in Fig 3.

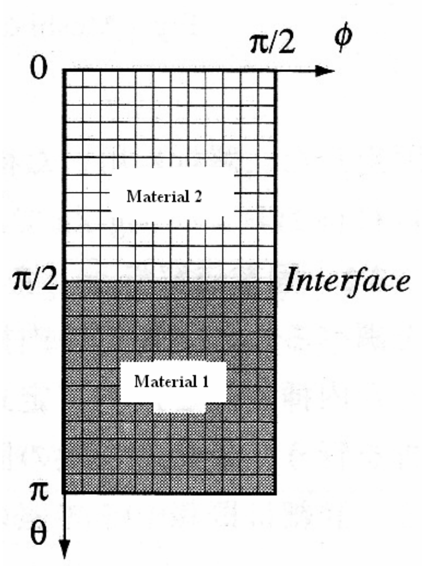

Fig. 3 Mesh model for FEM eigen analysis.

Many combinations of materials yielding the same value of Dundur's parameters generally exist. Hence, Young's modulus and Poisson's ratio, E_1 and ν_1 of material 1 are fixed, then E_2 and ν_2 of material 2 are determined for the given Dundur's parameters $\alpha - \beta$, by

$$E_2 = \frac{2G_1(1+\nu_2)}{\kappa} \tag{7}$$

$$\nu_2 = \begin{cases} 1 - \dfrac{m_2}{4} & \text{for plane strain} \\ \dfrac{4}{m_2} - 1 & \text{for plane stress} \end{cases} \tag{8}$$

where

$$G_1 = \frac{E_1}{2(1+\nu_1)} \tag{9}$$

$$\kappa = \frac{1 - \alpha + \alpha m_1 - \beta m_1}{1 - \alpha} \tag{10}$$

$$m_2 = \frac{m_1(\alpha + 1)}{1 - \alpha + \alpha m_1 - \beta m_1} \tag{11}$$

Here, the Dundur's parameters are employed to compare the contour map of the order of stress singularity for two-dimensional joints and three-dimensional joints.

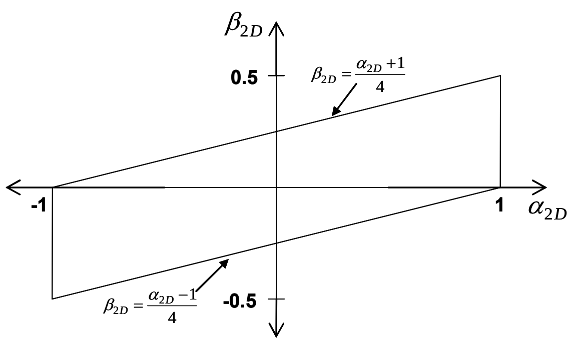

Fig. 4 Dundur's parameter plane.

3. Results and Discussion

The results in 3D joints are compared with those in 2D joints of the same cross section plane of 3D joints in the same material combination. The order of stress singularity in 2D joints is determined by using FEM eigen method. Afterwards, the contour map of the order of stress singularity on Dundur's composite plane is shown in Fig. 6 for the vertex point in 3D joints and for the apex in 2D joints. The zero boundary of singularity in 2D joints is presented by two lines, $\alpha = 0$ and $\beta = \alpha / 2$. In this study, the loci of the root of characteristic equations for the order of stress singularity are investigated by varying the value of α in the range $2\beta \le \alpha \le 1.0$ while holding β at the fixed value. In Table 1, the eigen value p for β being 0.1 is investigated precisely of various values of α from 0.2 to 1.0 by the three-dimensional FEM eigen analysis. The order of stress singularity $(\lambda = p - 1)$ for power-law singularity can be obtained directly as shown in Fig. 5. The same procedure is also used for various values of the Dundur's parameter β.

Table. 1 Eigen value p as $\beta = 0.1$

β=0.1		
α	p value	
	3D eigen FEM	2D eigen FEM
0.20	0.997093679	0.999212353
0.30	0.966312813	0.976054311
0.40	0.923317231	0.940251359
0.46	0.893478412	0.914796675
0.50	0.872330690	0.896616715
0.58	0.827632741	0.858036609
0.64	0.792357352	0.827575115
0.70	0.755764819	0.796062050
0.76	0.717924396	0.763622173
0.84	0.665492066	0.719002280
0.96	0.582069138	0.648962801
0.98	0.567511901	0.636877821

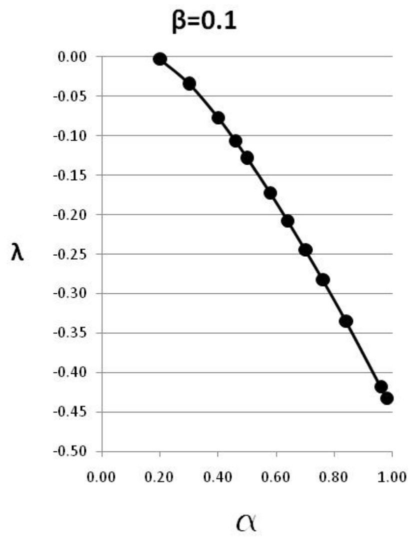

Fig. 5 The order of stress singularity as $\beta = 0.1$.

It can be seen that the order of stress singularities at the vertex point in three-dimensional bonded joins are larger than those for two-dimensional bonded joints.

4. Acknowledgement

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5. Conclusions

The order of stress singularity in a form of power-law singularity at the point on the stress singularity lines in three-dimensional bonded joints were investigated using the FEM eigen analysis. The contour map of the order of stress singularity in a form of power-law singularity for the vertex point in three-dimensional bonded joints were plotted on an ordinary Dundurs' composite plane $\alpha_{2D} - \beta_{2D}$ in plane strain condition. It can be seen that the order of stress singularity around the singular point on the stress singularity line in three-dimensional bonded joints was larger than that at the apex in 2D bonded joints.

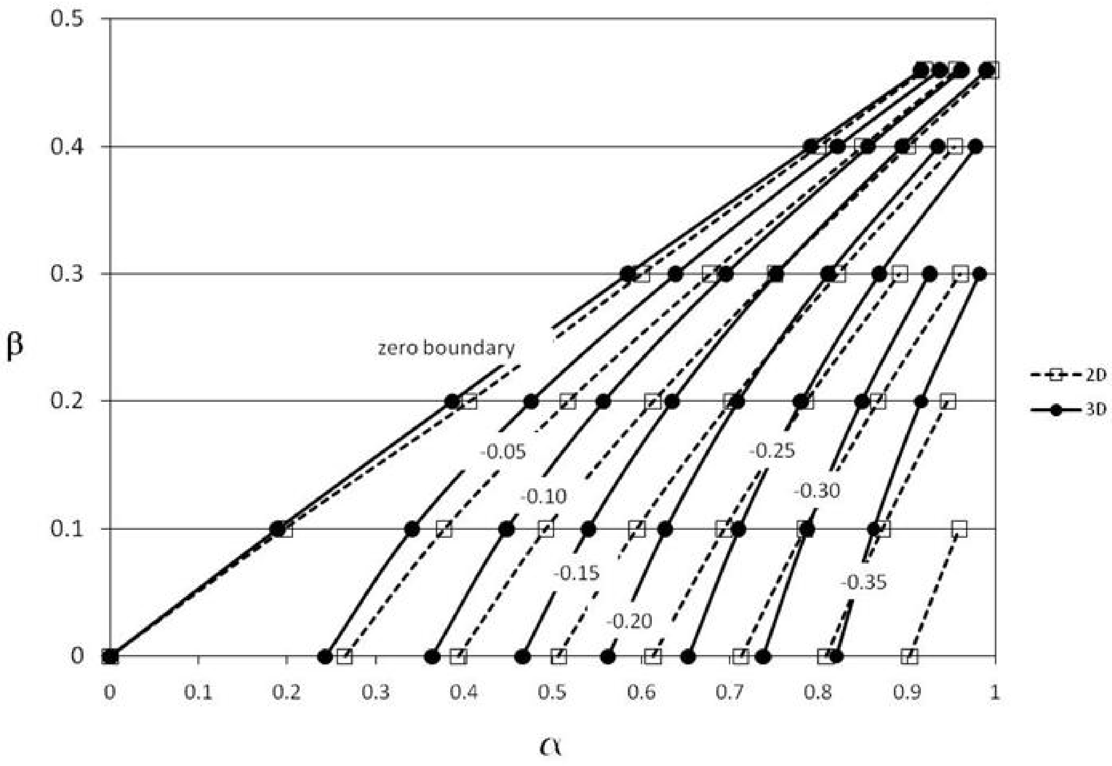

Fig. 6 The order of stress singularity on the Dundur's parameter plane $\alpha - \beta$ for the vertex point in three-dimensional bonded joints.

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