An experiment verification of the scaling law for buckling of cross-ply composite plates

สรสิทธิ์ อรัญพิทักษ์ ไพโรจน์ สิงหถนัดกิจ วริทธิ์ อึ๊งภากรณ์
ภาควิชาวิศวกรรมเครื่องกล คณะวิศวกรรมศาสตร์ จุฬาลงกรณ์มหาวิทยาลัย
เขตปทุมวัน กรุงเทพ 10330
โทร: 0-2218-6595 โทรสาร: 0-2252-2889 E-mail: Pairod.S@chula.ac.th

Sorasit Arunpitak, Pairod Singhatanadgid, Variddhi Ungbhakorn
Department of Mechanical Engineering, Faculty of Engineering, Chulalongkorn University,
Bangkok, 10330, Thailand
Tel: 0-2218-6595 Fax: 0-2252-2889 E-mail: Pairod.S@chula.ac.th

Abstract

In this study, the scaling law for buckling of cross-ply composite plates is verified by experiment method. A unidirectional compressive-loading test frame was designed to accommodate the buckling test. Besides applying a uniform compressive load, the test frame is able to apply simply supported boundary conditions on a rectangular specimen. A dial indicator was placed on the specimen to measure the maximum deflection. Buckling loads were determined from a plot of applied compressive vs. out-of-plane displacement using inflection point method. To verify the scaling law, specimens were classified as prototype and model. The measured buckling loads of the model were then substituted in the derived scaling law to predict the buckling loads of the prototype, which were compared to the measured buckling load. Buckling loads of the prototype determined from both approaches agree with each other very well for specimens with aspect ratios of 1 and 1.5. A specimen with aspect ratio of 2 has fairly high percent of discrepancy. Imperfections of experiment setting and the technique used to identify buckling load are probably the causes of discrepancy. More experiments are required in order to reach a firm conclusion.

1. Introduction

Laminate composites are widely used for components in mechanical and aerospace engineering applications due to their high specific stiffness (stiffness per unit density) and high specific strength (strength per unit density). Buckling of composite plates is in the interest of many researchers in the past decades. A lot of analytical and numerical studies concerning of buckling of composite plates are available in the literatures [1-5]. Also, a lot of experiments were also conducted to verify those analytical and numerical predictions. However, high discrepancy between measured and theoretical buckling loads was usually reported. Chai *et al.* [6] investigated the laminated plates under uniaxial loading using LVDT and strain gage to measure the out-of-plane deflection and in-plane strain, respectively. The buckling loads from experiment correlated well with finite element solutions. Discrepancies between -7% and 11% of the experimental buckling loads were reported. Tuttle *et al.* [7] performed experiments on the laminated plates subjected to biaxial loading using specimens with four stacking sequences and three aspect ratios. They found rather large discrepancies, in some cases up to 35%, in buckling loads between measurement and theoretical prediction. Most of the study reported the inconsistency between theoretical and experimental buckling loads was probably initiated from the imperfections of specimens, boundary conditions, and loading configurations. Thus, it is necessary to include the effects of imperfections into the mathematical model, if a better prediction is desired. This is where the similitude method appears as an important tool to predict the buckling behavior of the imperfect panels more accurately. Similitude theory can be roughly stated to be a branch of science concerned with sufficient and necessary conditions of similarity among phenomena. If such similarity conditions can be found among parameters of the model and prototype, then the scaled replica can be built to duplicate the behaviors of the full-scaled system, and the results from the model experiments can be employed to predict the behavior of the prototype which has complete similarities with the test model.

The similitude transformation for a stability problem of symmetrically cross-ply laminated plates buckling has been established by Singhatanadgid *et al.* [8]. The scaling laws were verified numerically using available closed-form and numerical solutions. In the present study, the scaling law is validated again by performing experiments on both models and prototypes. Then, the buckling loads of the models are substituted into the scaling law to predict the buckling loads of the prototypes. The accuracy of the scaling law is determined by comparing the buckling load of the prototypes from the derived scaling law with the experimental buckling load.

2. Scaling law for buckling problems

When a rectangular thin plate is subjected to a uniaxial in-plane compressive load, buckling occurs when the applied load is sufficiently high. The classical buckling differential equation of cross-ply composite plates can be written as; [9]

$$D_{11}w,_{xxxx} + 2(D_{12}+2D_{66})w,_{xxyy} + D_{22}w,_{yyyy} + \bar{N}_x w,_{xx} = 0 \qquad (1)$$

The scaling laws for buckling of cross-ply composite plates are derived from the governing equation by writing the governing equation for the model and prototype, then writing the similitude invariant term which leads to the scaling law. Let the variables of the prototype be related to those of the model through the similitude scaling factors (C_x, C_y, $C_{w....}$) as follows.

$$x_p = C_x x_m \quad , \quad y_p = C_y y_m \quad , \quad w_p = C_w w_m$$
$$\left(D_{ij}\right)_p = C_{Dij}\left(D_{ij}\right)_m \quad , \quad \text{and} \quad \left(\bar{N}_x\right)_p = C_{\bar{N}x}\left(\bar{N}_x\right)_m$$

where subscripted "p" refers to "prototype" and subscripted "m" refers to "model"

By applying similitude transformation to Eq.(1), the following necessary conditions for the models to behave exactly as the prototype are obtained:

$$\frac{C_{D11}}{C_x^4} = \frac{C_{D12}}{C_x^2 C_y^2} = \frac{C_{D66}}{C_x^2 C_y^2} = \frac{C_{D22}}{C_y^4} = \frac{C_{\bar{N}x}}{C_x^2} \qquad (2)$$

Let the prototype and model be related with complete geometric similarity, therefore $C_x = C_y = C_a = C_b$, where a and b are plate's height and width, respectively. Thus, Eq. (2) can be rewritten as:

$$C_{\bar{N}x} = \frac{C_{D11}}{C_b^2} = \frac{C_{D12}}{C_b^2} = \frac{C_{D22}}{C_b^2} = \frac{C_{D66}}{C_b^2} \qquad (3)$$

For complete similarity between the prototype and its model, it is required that the scaling factors of all laminate flexural stiffnesses must be equal, e.g.

$$C_{D11} = C_{D12} = C_{D22} = C_{D66} \qquad (4)$$

Let the scaling factors of the flexural stiffnesses be equal to C_{stiff}, then the above equations yield the following similitude invariant for the symmetric laminated plates subjected to combined in-plane loads:

$$\frac{C_{\bar{N}x} C_b^2}{C_{stiff}} = 1 \qquad (5)$$

which gives the following scaling law:

$$\bar{N}_{xp} = \bar{N}_{xm} C_{stiff} \frac{b_m^2}{b_p^2} \qquad (6)$$

The complete similitude is achieved when the prototype and model have complete geometric similarity, that is $C_a = C_b$. It is also required that the scaling factors of all laminate flexural stiffnesses must be equal. The scaling law was validated with the theoretical solution in Ref [8]. The exact agreement between results from the scaling law and those from the closed-form solution is obtained when models and prototypes have complete similarities.

In this study, the scaling law in Eq.(6) is experimentally verified on a custom-made compressive test frame. Two sets of specimens, i.e. model and prototype, are prepared and tested for buckling loads. The buckling loads of the models, $\bar{N}_{xm}$, are then substituted into Eq.(6) to determine the buckling loads of the prototypes, $\bar{N}_{xp}$. The accuracy of the scaling law is determined by comparing $\bar{N}_{xp}$ from the scaling law with that of from the experiment on the prototype.

3. Experiment Setup

Two sets of cross-ply composite plates, i.e. model and prototype, were tested under a uniaxial in-plane compression to determine the bucking loads. The test panels were mounted on a specially designed loading frame and subjected to compressive loading as shown in Fig 1. Compressive loads were vertically applied by a hydraulic cylinder onto the movable crosshead. The movable crosshead can be moved vertically on four circular columns with supports from linear bearings embedded in the crosshead. Applied compressive load was monitored by a 10-ton load cell placed on top of the crosshead. The simple support boundary conditions were simulated on four edges of the specimen as shown in Fig. 2. The top and bottom sides of the specimen were placed into a slot of a circular slotted rod, which was placed into a semi-circular slot of the crossheads, Fig. 2(a). With this configuration, the test panels are allowed to rotate, but restrained to move in the out-of-plane direction. For the side supports, two knife-edge supports were placed on the specimen to simulate the simple support as shown in Fig. 2(b). Similar to the top and bottom supports, side edges of the specimen can rotate, but cannot move out of plane. The out-of-plane displacement was measured using a dial indicator.

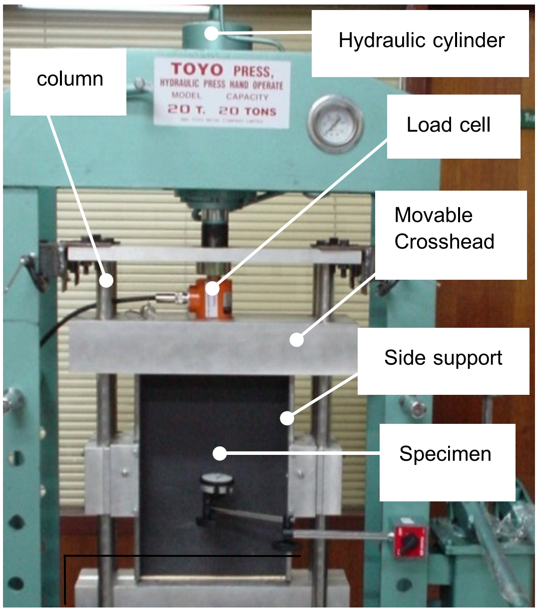

Fig. 1 The compression test frame for buckling experiment

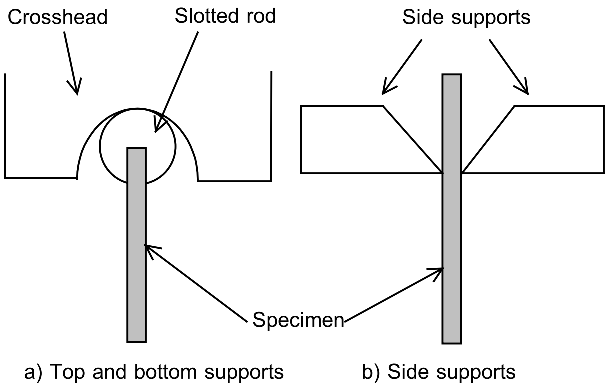

Fig. 2 The schematic of simply supported boundary condition

During the experiments, compressive load is applied and plotted with its respective out-of-plane displacement measured from a dial indicator. In the pre-buckling region, slope of the plot between applied loads vs. out-of-plane displacements is steeper than that of the post-buckling region, i.e. the out-of-plane displacement induced in the specimen is low. After the critical buckling load, a small increasing of compressive load initiates high out-of-plane displacement. A tropical plot of an applied load and out-of-plane displacement is shown in Fig. 3. The critical buckling load, $\bar{N}_x$, is determined from the inflection point or the point of least slope on the load-displacement curve [10].

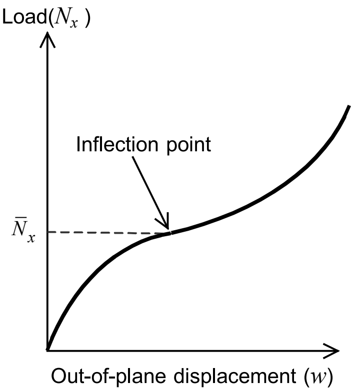

Fig. 3 Experimental buckling load from a plot of N_x vs w

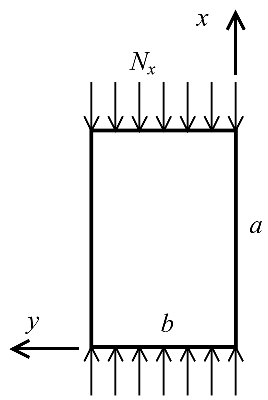

Fig. 4 A rectangular plate subjected to a uniaxial in-plane load

Specimens in this study are comprised of six rectangular laminated composite plates with the dimension as shown in Fig. 4. The panels are made from graphite/epoxy prepreg with the stacking sequence of $[0/90]_{2s}$. The models width (b) is 24 cm with three aspect ratios of 1, 1.5, and 2, i.e. the height of the panel (a) is 24, 36, and 48 cm, respectively. Three prototypes are panels with the width of 18 cm. Since, The scaling law requires geometric similarity between model and prototype so that they have to have the same aspect ratios, i.e. the heights of the prototypes are 18, 27, and 36 cm, respectively.

4. Experiment result

The load-deflection curves of the prototypes are shown in Fig. 5-7. It is observed that out-of-plane displacement is detected as soon as compressive load is applied. This phenomenon indicates that there are imperfections of the loading method, boundary condition, or the specimen itself. Load-deflection curves of the models are similar to those of the prototypes.

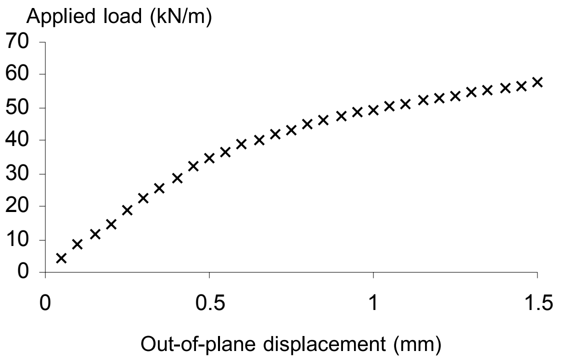

Fig. 5 Load-deflection curve for prototype size 18 x 18 cm^2

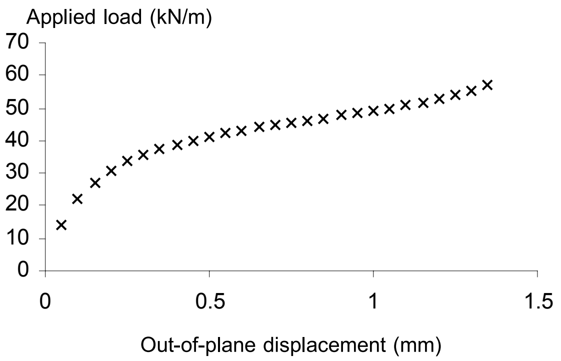

Fig. 6 Load-deflection curve for prototype size 27 x 18 cm^2

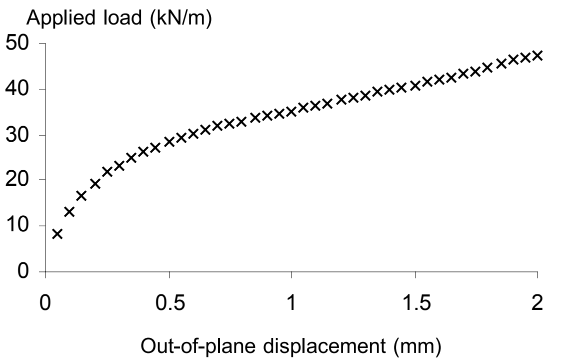

Fig. 7 Load-deflection curve for prototype size 36 x 18 cm^2

From the load-displacement curves, the buckling loads are determined from the inflection point, which is the point of least slope on the curve. These experimental buckling loads are shown in column 2 and 4 of Table 1 for models and prototypes, respectively. Although, the resolution of the load cell used in this experiment is as high as 0.01 kN, the experimental buckling loads are reported with a resolution of 1 kN because the data reduction processes depend on human's judgment more than mathematical technique. The first column of Table 1 is the aspect ratio of the specimens. The third column presents the buckling loads of prototypes determined from the derived scaling law, Eq. (6). In this study, both models and prototypes have the same stacking sequence so D_{ij} for both systems are identical or C_{stiff} = 1. The scaling law for models and prototypes with specimen width of 24 and 18 cm, respectively, is reduced to

$$\bar{N}_{xp} = \bar{N}_{xm}\frac{24^2}{18^2} = \frac{16}{9}\bar{N}_{xm} \qquad (7)$$

The experimental buckling loads of the models shown in column 2 are substituted in Eq.(7) to predict the buckling of the prototypes, as shown in the 3rd column. Predicted buckling loads of the prototypes are verified by the experiment results, shown in column 4. The discrepancies of the scaling buckling loads are presented in the last column in term of percent discrepancy which is calculated according to:

$$\%Disc. = \frac{\bar{N}_{xp} - \bar{N}_{xp}^{Exp.}}{\bar{N}_{xp}^{Exp.}} \times 100\% \qquad (8)$$

Table 1. Buckling loads in kN/m of the tested panels

Aspect Ratio	Model, Exp $\bar{N}_{xm}$ (b = 24 cm)	Prototype (b = 18 cm)		
		Scaling Law, $\bar{N}_{xp}$	Exp. $\bar{N}_{xp}^{Exp.}$	% Disc.
1	27	48.0	48	0
1.5	23	40.9	42	-2.6
2	22	39.1	32	22.2

5. Discussion and Conclusion

The load-displacement curves of all specimens clearly show buckling behavior and imperfections of the test setup. There is an apparent change of slope between pre-buckling and post-buckling regions. Imperfection of the experiment setup is detected from an out-of-plane displacement observed at a very low compressive load. In practice, it is fairly difficult to indicate the inflection point for some cases of the experiments because there is more than one inflection point, i.e. there is a region of constant slope on the load-displacement curve. In this study, buckling load is selected from the lowest applied load in the inflection region.

Comparisons of the predicted buckling loads from the scaling law with the experimental buckling loads of the prototypes are very good for specimen with aspect ratio of 1 and 1.5. Moderately high percent of discrepancy is obtained for specimens with aspect ratio of 2. The discrepancy is possibly generated by the imperfections of each experiment. Although, the scaling law is able to account for imperfections, it is required that the imperfections of the model and the prototype have similarities, as well. In this study, there was an effort to keep the imperfection of both systems as low as possible, although it is impossible to measure the imperfection for each experiment. This is probably the cause of discrepancy of the scaling buckling load and the experimental buckling load shown in the last column of Table 1. Nevertheless, it is common to have fairly high percent of discrepancy in buckling experiment of composite plates, according to the past studies [6,7].

One of the difficulties in this experiment is the method of identifying buckling point. As mention previously, the inflection point in some cases is doubtful. Other technique such as average strain should be explored as a method of identifying the buckling load. Some more experiments should be performed with additional measurement techniques.

In conclusion, the scaling law of buckling of cross-ply laminated plates is verified experimentally. Two sets of $[0/90]_{2s}$ graphite/epoxy specimens were tested for buckling loads. The scaling buckling loads are well compared to the experimental buckling loads for specimens with aspect ratios of 1 and 1.5. Fairly high percent of discrepancy is obtained in case of specimen with aspect ratio of 2. The imperfections of the experiments and the inflection point technique used to identify buckling load are probably two causes of discrepancy. More experiments are required in order to reach a firm conclusion.

6. Acknowledgement

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