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A Sliding mode Model Following Control (SMFC) for an Electrohydraulic Position Servo Control Systems

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Abstract

A Sliding mode Model Following Control or SMFC strategy for an electrohydraulic position servo control system is presented. The SMFC algorithm uses the combination of model following control and sliding mode control to improve the dynamics response for command tracking. The SMFC structure consists of a double-integrator, a feedforward path from the input command and a reference model. The control function is derived to guarantee the existence of a sliding mode. Furthermore, the chattering in the control signal is suppressed by replacing the sign function in the control function with a smoothing function. Simulation results illustrate that the purposed approach gives a significant improvement on the tracking performances. Its can achieve accurate servo tracking in the presence of plant parameter variation and external load disturbances.

1. Introduction

Processes requiring large driving forces or torques are often actuated by hydraulic servo system. The dynamic characteristics of such systems are complex and highly nonlinear due to the flow pressure relationship of the hydraulic components. For a practical control system, it is usually desired to have a fast accurate response with a small overshoot. Due to the nonlinear dynamic property of hydraulic servo valves, it is not easy to design the control system of hydraulic position servos with a simple linear controller.

In certain case, a variable structure control (VSC) is invariant to system parameter variations and disturbances when the sliding mode occurs [1-3]. Because of its simple construction, high reliability and fast response without overshoot. The sliding mode operation results in a control system that is robust to model certainties, parameter variations and disturbances. Although the conventional VSC approach has been applied successfully in many applications, but it may result in a steady state error when there is load disturbance in it. In order to improve the problem, the integral variable structure control (IVSC) is presented in [4-5] combines and integral controller with the variable structure control. The IVSC approach comprises an integral controller for achieving a zero steady state error under step input and a VSC for enhancing the robustness. However, its performance when changing,e.g., ramp command input, the IVSC gives a steady state error. The Modified Integral Variable Structure Control (MIVSC), proposed in [6], uses a double-integral action to solve this problem and improve the dynamics response for command tracking. Although, the MIVSC method can give a better tracking performance than the IVSC method does at steady state, its performance during transient period needs to be improved.

In this paper, The design and simulation of an electrohydraulic position servo control systems using the Sliding mode Model Following Control or SMFC approach is described. The advantage of this approach is that the error trajectory in the sliding motion can be prescribed by the design. Also, it can achieve a rather accurate servo tracking and is fairly robust to plant parameter variations and external load disturbances. As a simulation results, the tracking performance can be remarkably improved and is fairly robust to plant parameter variations and external load disturbances. The design of a SMFC system involves : 1) the choice of the control function to guarantee the existence of a sliding motion and 2) the determination of the switching function and the integral control gain such that the system has desired properties.

2. Design of SMFC systems

The structure of SMFC system is shown in Fig. 1. It combines the conventional VSC with a double-integral compensator, a feedforward path from the input command, a reference model and a comparator.

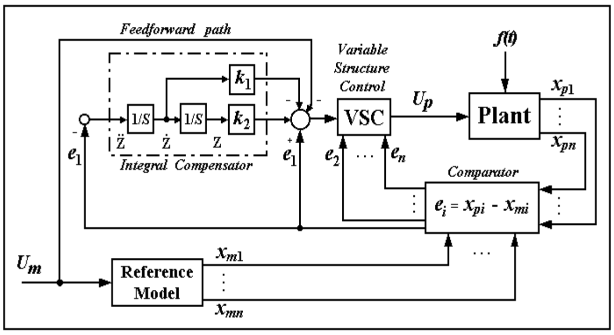

Fig. 1. The structure of SMFC system.

Let the plant be described by the following equation :

$$\dot{x}_{pi} = x_{p(i+1)} \qquad ;i=1,\ldots,n\text{-}1 \tag{1a}$$

$$\dot{x}_{pn} = -\sum_{i=1}^{n} a_{pi}x_{pi} + b_pU_p - f(t) \tag{1b}$$

where a_{pi} and b_p are the plant parameters;
$f(t)$ are disturbances;
r is the reference input;
x_1 is the output;
U_p is the control input of the plant.

The reference model is represented by

$$\dot{x}_{mi} = x_{m(i+1)} \qquad ;i=1,\ldots,n\text{-}1 \tag{2a}$$

$$\dot{x}_{mn} = -\sum_{i=1}^{n} a_{mi}x_{mi} + b_mU_m \tag{2b}$$

where U_m is the input command of the system.

Defining $e_i = x_{pi} - x_{mi}$ $;(i=1,\ldots,n)$ and subtracting (2) from (1), the error differential equation is

$$\dot{e}_i = e_{i+1} \qquad ;i=1,\ldots,n\text{-}1 \tag{3a}$$

$$\dot{e}_n = -\sum_{i=1}^{n} a_{pi}e_i - \sum_{i=1}^{n}(a_{mi}-a_{pi})x_{mi} + b_mU_m - b_pU_p - f(t)\cdot \tag{3b}$$

Using the FIVSC approach [7] to the error dynamics in order to synthesise the control signal, U_p and assuming the asymptotic divergence of the error to zero, the SMFC system can be described as

$$\ddot{z} = -e_1 \tag{4a}$$

$$\dot{e}_i = e_{i+1} \qquad ;i=1,\ldots,n\text{-}1 \tag{4b}$$

$$\dot{e}_n = -\sum_{i=1}^{n} a_{pi}e_i - \sum_{i=1}^{n}(a_{mi}-a_{pi})x_{mi} + b_mU_m - b_pU_p - f(t) \tag{4c}$$

where U_p is the control function.

The switching function, σ is given by

$$\sigma = c_1(x_1 - K_1\dot{z} - K_2z - r) + \sum_{i=2}^{n} c_ie_i \quad ;C_i\text{>}0\text{=constant},\ C_n\text{=}1. \tag{5}$$

The control signal can be determined as follows. From (4) and (5), we have

$$\dot{\sigma} = -c_1(K_1\ddot{z} + K_2\dot{z}) + \sum_{i=2}^{n} c_{i-1}e_i - \sum_{i=1}^{n} a_{pi}e_i + \sum_{i=1}^{n}(a_{mi}-a_{pi})x_{mi} - b_mU_m + b_pU_p - f(t). \tag{6}$$

Let $a_{pi} = a_{pi}{}^0 + \Delta a_{pi}$ $;i=1,\ldots,n$

and $b_p = b_p{}^0 + \Delta b_p$ $;b_p{}^0 > 0,\ \ \Delta b_p > -\ b_p{}^0$

where $a_{pi}{}^0$ and $b_p{}^0$ are nominal values;
Δa_{pi} and Δb_p are the associated variations.

Let the control function be

$$U_p = U_{eq} + U_s \tag{7}$$

where the so called equivalent control U_{eq} is defined as the solution of (6) under the condition where there is no disturbances and no parameter variations, that is $\dot{\sigma} = 0$, $f(t) = 0$, $a_{pi} = a_{pi}{}^0$, $b_p = b_p{}^0$, $U_p = U_{eq}$.

This condition results in

$$U_{eq} = \Big\{-c_1(K_1\ddot{z} + K_2\dot{z}) - \sum_{i-2}^{n-1} c_{i-1}e_i + \sum_{i=1}^{n-1} a_{pi}^0 e_i - \sum_{i=1}^{n}(a_{mi}-a_{pi})x_{mi} + b_mU_m$$
$$+ (c_{i-1} - a_{pn}^0)\Big[c_1(e_1 - K_1\dot{z} + K_2z - U_m) + \sum_{i-2}^{n-1} c_ie_i\Big]\Big\}/b_p^0. \tag{8}$$

The function U_s, is employed to eliminate the influence due to Δa_{pi}, Δb_p and $f(t)$.

It is required to guarantee the existence of the sliding mode. This function is constructed as

$$U_s = \varphi_1(e_1 - K_1\ddot{z} - K_2z - U_m) + \sum_{i=2}^{n} \varphi_ie_i + \varphi_{n+1}\cdot \tag{9}$$

The condition for the existence and reachability of a sliding mode is known to be

$$\sigma\dot{\sigma} \langle\ 0\ . \tag{10}$$

Substitute (7) and (9) into (6), to obtain

$$\dot{\sigma}\sigma = [-\Delta a_{p1} + a_{p1}^0\Delta b_p/b_p^0 + c_1(c_{n-1} - a_{pn}^0)(1 + \Delta b_p/b_p^0) + b_p\varphi_1](e_1 - K_1\dot{z} - K_2z - U_m)\sigma$$
$$+ \sum_{i=2}^{n-1}\{[-\Delta a_{pi} + a_{pi}^0\Delta b_p/b_p^0 - c_{i-1}\Delta b_p/b_p^0 + c_i(c_{n-1} - a_{pn}^0)(1 + \Delta b_p/b_p^0) + b_p\varphi_i]e_i\sigma\}$$
$$+ [-\Delta a_{pn} + (c_{n-1} - a_{pn}^0) + b_p\varphi_n]e_n\sigma \ + [N + b_p\varphi_{n+1}]\sigma \tag{11}$$

where $N = -(K_1\dot{z} + K_2z)(\Delta a_{p1} - a_{p1}^0\Delta b_p/b_p^0) + \Delta b_p/b_p^0[c_1(K_1\ddot{z} + K_2\dot{z})]$
$+ [-\sum_{i=1}^{n}(a_{mi} - a_{pi}^0)x_{mi} + b_mU_m]\Delta b_p/b_p^0 - f(t).$

In order for (10) to be satisfied, the following conditions must be met,

$$\varphi_i = \begin{cases} \alpha_i\ \langle\ \mathrm{Inf}[\Delta a_{pi} - a_{pi}^0\Delta b_p/b_p^0 + c_{i-1}\Delta b_p/b_p^0 - c_i(c_{n-1} - a_{pn}^0)(1 + \Delta b_p/b_p^0]/b_p \\ \beta_i\ \rangle\ \mathrm{Sup}[\Delta a_{pi} - a_{pi}^0\Delta b_p/b_p^0 + c_{i-1}\Delta b_p/b_p^0 - c_i(c_{n-1} - a_{pn}^0)(1 + \Delta b_p/b_p^0]/b_p \end{cases} \tag{12a}$$

where $i=1,\ldots,n\text{-}1$, $c_0 = 0$

$$\varphi_n = \begin{cases} \alpha_n \ \langle \ \mathrm{Inf}[\varDelta a_{pn} + a_{pn}^0 - c_{n-1}]/b_p \\ \beta_n \ \rangle \ \mathrm{Sup}[\varDelta a_{pn} + a_{pn}^0 - c_{n-1}]/b_p \end{cases} \tag{12b}$$

and
$$\varphi_{n+1} = \begin{cases} \alpha_{n+1} \ \langle \ \mathrm{Inf}[-N]/b_p \ . \\ \beta_{n+1} \ \rangle \ \mathrm{Sup}[-N]/b_p \end{cases} \tag{12c}$$

Let φ_i, $i=1,\dots,n+1$, be chosen as $\varphi_i = \alpha_i = -\beta_i$.

Finally, the control can be represented as

$$\begin{aligned} U_p = \Big\{ & -c_1(K_1\ddot{z} + K_2\dot{z}) - \sum_{i=2}^{n-1} c_{i-1}e_i + \sum_{i=1}^{n-1} a_{pi}^0 e_i - \sum_{i=1}^{n}(a_{mi} - a_{pi}^0)x_{mi} + b_m U_m \\ & + (c_{n-1} - a_{pn}^0)\left[c_1(e_1 - K_1\dot{z} - K_2 z - U_m) + \sum_{i=2}^{n-1} c_i e_i\right]\Big\}/b_p^0 \\ & + \left(\varphi_1 |e_1 - K_1\dot{z} - K_2 z - U_m| + \sum_{i=2}^{n} \varphi_i |e_i| + \varphi_{n+1}\right) sign(\sigma) \end{aligned} \tag{13}$$

where

$$\varphi_i \ \langle \ -\mathrm{Sup}\left|[\varDelta a_{pi} - a_{pi}^0 \varDelta b_p / b_p^0 + c_{i-1}\varDelta b_p / b_p^0 - c_i(c_{n-1} - a_{pn}^0)(1 + \varDelta b_p / b_p^0)]/b_p\right|$$

$$\varphi_n \ \langle \ -\mathrm{Sup}\left|[\varDelta a_{pn} + a_{pn}^0 - c_{n-1}]/b_p\right|$$

and $\varphi_{n+1} \ \langle \ -\mathrm{Sup}\left|[N]/b_p\right|$ ·

The transfer function when the system is on the sliding surface can be shown as

$$H(s) = \frac{E_1(s)}{U_m(s)} = \frac{s^2 c_1}{s^{n+1} + c_{n-1}s^n + \ldots + c_1 K_1 s + c_1 K_2} \ . \tag{14}$$

Using the final value theorem, it can be shown from (14), that the steady-state tracking error due to a ramp command input is zero. The transient response of the system can be determined by suitably selecting the poles of the transfer function(14).

Let
$$S^{n+1} + \alpha_1 S^n + \ldots + \alpha_{n-1} S + \alpha_n = 0 \tag{15}$$

be the desired characteristic equation(closed-loop poles), the coefficient C_1, C_2 and K_1, K_2 can be obtained by

$$C_{n-1} = \alpha_1,\ C_1 = \alpha_{n-2},\ K_1 = \alpha_{n-1}/C_1 \text{ and } K_2 = \alpha_n/C_1.$$

Normally, the sign function sign (σ) given by (13), will give rise to chattering in the control signal. In order to reduce the chattering, the sign function can be replaced by the continuous function, given by

$$M_\delta(\sigma) = \frac{\sigma}{|\sigma| + \delta_0 + \delta_1|\ddot{z}|} \tag{16}$$

where $\delta = \delta_0 + \delta_1|\ddot{z}|$

and δ_0 and δ_1 are positive constants.

3. Dynamic modeling of an electrohydraulic position servo control systems

The block diagram of the electrohydraulic position servo control systems to be studied is shown is Fig. 2. The relation between the valve displacement X_v and the flow rate Q_L is described as [8]

$$Q_L = X_v K_j \sqrt{P_s - \mathrm{sign}(X_v)P_L} = X_v K_s \tag{17}$$

where K_j is a constant for a specific hydraulic motor;
P_s is the supply pressure;
P_L is the load pressure and
K_s is the valve flow gain that varies under different operating points.

The flow continuity property of the motor chamber yields

$$Q_L = D_m\omega_c + K_{ce}P_L + \left(\frac{V_t}{4\beta}\right)\dot{P}_L \tag{18}$$

where D_m is the volumetric displacement;
K_{ce} is the total leakage coefficient;
V_t is the total volume of the oil;
β is the bulk modulus of the oil and
ω_c is the velocity of the motor shaft.

The torque balance equation for the motor is given by

$$D_m P_L = J\dot{\omega}_c + B_m\omega_c + \dot{T}_L \tag{19}$$

where B_m is the viscous damping coefficient;
J is the inertia of the motor and
T_L is the load disturbance.

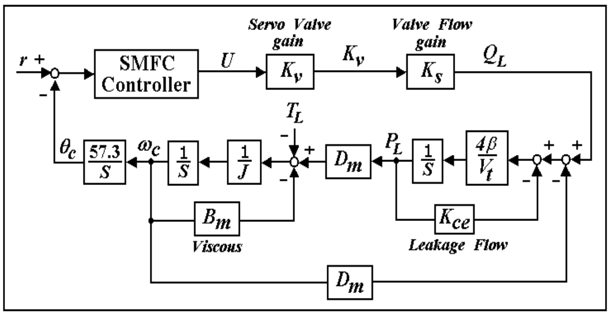

Fig. 2. The electrohydraulic position servo systems using SMFC controller.

4. The SMFC system for an electrohydraulic systems

The nominal values of the electrohydraulic parameters and the SMFC controller are listed in Table. 1 and Table. 2, respectively. Based on the block diagram as shown in Fig. 2, by combining (17)-(19), the following set of state equations can be obtained :

$$\dot{x}_{p1} = x_{p2} \tag{20a}$$

$$\dot{x}_{p2} = x_{p3} \tag{20b}$$

$$\dot{x}_{p3} = -a_{p1}x_{p1} - a_{p2}x_{p2} - a_{p3}x_{p3} + b_p U_p - f(t) \tag{20c}$$

where

$$a_{p2} = \frac{4\beta}{V_t}\frac{D_m^2}{J} + \frac{4\beta}{V_t}\frac{B_m}{J}K_{ce},$$

$$a_{p3} = \frac{B_m}{J} + \frac{4\beta}{V_t}K_{ce},$$

$$b_p = 57.3 K_v K_s \frac{4\beta}{V_t}\frac{D_m}{J},$$

$$f(t) = 57.3\frac{4\beta}{V_t}\frac{K_{ce}}{J}T_L + 57.3\frac{1}{J}\dot{T}_L.$$

The system will be used in the design of SMFC system.

The reference model is chosen as

$$\dot{x}_{m1} = x_{m2} \tag{21a}$$

$$\dot{x}_{m2} = x_{m3} \tag{21b}$$

$$\dot{x}_{m3} = -a_{m1}x_{m1} - a_{m2}x_{m2} - a_{m3}x_{m3} + b_m U_m \tag{21c}$$

Defining $e_i = x_{pi} - x_{mi}$;(i= 1,2,3), the SMFC system can be represented as

$\dot{e}_1 = e_2$, $\dot{e}_2 = e_3$ and

$$\begin{aligned} \dot{e}_3 = &-a_{p1}e_1 - a_{p2}e_2 - a_{p3}e_3 + (a_{m1} - a_{p1})x_{m1} \\ &+ (a_{m2} - a_{p2})x_{m2} + (a_{m3} - a_{p3})x_{m1} - b_m U_m + b_p U_p - f(t) \end{aligned} . \tag{22}$$

Following the design procedure we have the control law to simulate as

$$\begin{aligned} U_P = \{ & c_1(K_1\ddot{z} + K_2\dot{z}) + a_{p1}^0 e_1 + a_{p2}^0 e_2 - [(a_{m1} - a_{p1}^0)x_{m1} \\ & + (a_{m2} - a_{p2}^0)x_{m2} + (a_{m3} - a_{p3}^0)x_{m3} + b_m U_m)] \\ & + (c_2 - a_3^0)[c_1(e_1 - K_1\dot{z} - K_2 z - r) + c_2 e_2]\} / b^0 \\ & + (\varphi_1|e_1 - K_1\dot{z} - K_2 z - r| + \varphi_2|e_2| + \varphi_3|e_3| + \varphi_4)M_\delta(\sigma) \end{aligned} \tag{23}$$

where

$\varphi_1 \langle -\text{Sup}\left|[\Delta a_{p1} - a_{p1}^0 \Delta b_p / b_p^0 - c_1(c_2 - a_{p3}^0)(1 + \Delta b_p / b_p^0)] / b_p\right|$

$\varphi_2 \langle -\text{Sup}\left|[\Delta a_{p2} - a_{p2}^0 \Delta b_p / b_p^0 + c_1 \Delta b_p / b_p^0 - c_2(c_2 - a_{p3}^0)(1 + \Delta b_p / b_p^0)] / b_p\right|$

$\varphi_3 \langle -\text{Sup}\left|[\Delta a_{p3} + a_{p3}^0 - c_2] / b_p\right|$ and $\varphi_4 \langle -\text{Sup}\left|[N] / b_p\right|$.

The switching function, σ from (5), is

$$\sigma = c_1(e_1 - K_1\dot{z} - K_2 z - r) + c_2 e_2 + e_3 \quad ; r = U_m. \tag{24}$$

By considering the operation points, one assumes the range of the plant parameter variables to be

$$|\Delta a_{p1}| < 50\ \%\ a_{p1}^0,\ |\Delta a_{p2}| < 50\ \%\ a_{p2}^0,\ |\Delta a_{p3}| < 50\ \%\ a_{p3}^0,$$
$$|\Delta b_p| < 50\ \%\ b_p^0 \text{ and } |N| < 3{,}000.$$

Thus, from (22), the gain φ_1, φ_2 and φ_3 must be chosen to satisfy the following inequalities :

$$\varphi_1 < -0.0658,\ \varphi_2 < -0.0026,\ \varphi_3 < -0.0035$$
$$\text{and } \varphi_4 < -0.025.$$

Also, based on simulations, one possible set of the switching gains can be chosen as

$$\varphi_1 < -0.6,\ \varphi_2 < -0.00015,\ \varphi_3 < -0.004$$
$$\text{and } \varphi_4 < -0.014.$$

The robustness of the proposed SMFC approach against large variations of plant parameters and external load disturbances has been simulated for demonstration.

Table 1. System parameters of an electrohydraulic for simulation.

Parameter	Value	Dimension
K_s	$0.03 \times \sqrt{P_s - \text{sign}(X_v)P_L}$	in^2/s
P_s	2,000	psi
β	50,000	psi
V_t	2.0	in^3
K_{cec}	0.001	$in^3/s/psi$
D_m	1.0	in^3/rad
J	0.5	$in\text{-}lb\text{-}s^2/rad$
B_m	75	in-lb-s/rad
K_v	20	in/V

Table 2. Parameters of SMFC controller.

Symbol	Value
λ_1	$-11.54+22.93i$
λ_2	$-11.54-22.93i$
λ_3	-18.46
λ_4	-5.71
C_1	3645
C_2	158.83
K_1	34.75
K_2	121.06
φ_1	-0.6
φ_2	-0.00015
φ_3	-0.004
φ_4	-0.014
a_{m1}	12,000
a_{m2}	1,200
a_{m3}	36
b_m	12,000
a_{p2}^0	9,654.57
a_{p3}^0	2,492
b_p^0	88,967.72
δ_0	8
δ_1	80

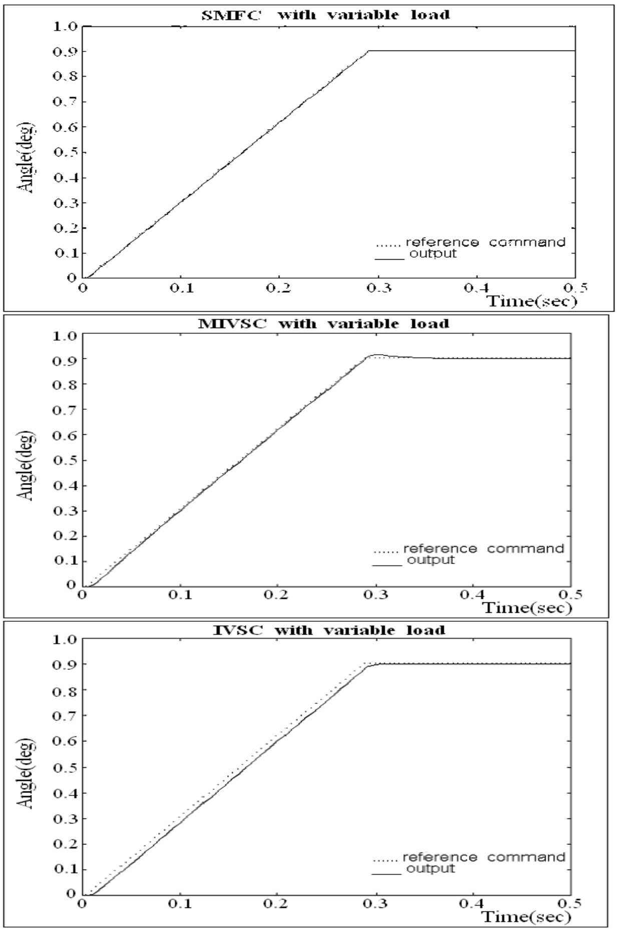

Fig. 3. Comparison of ramp position tracking.

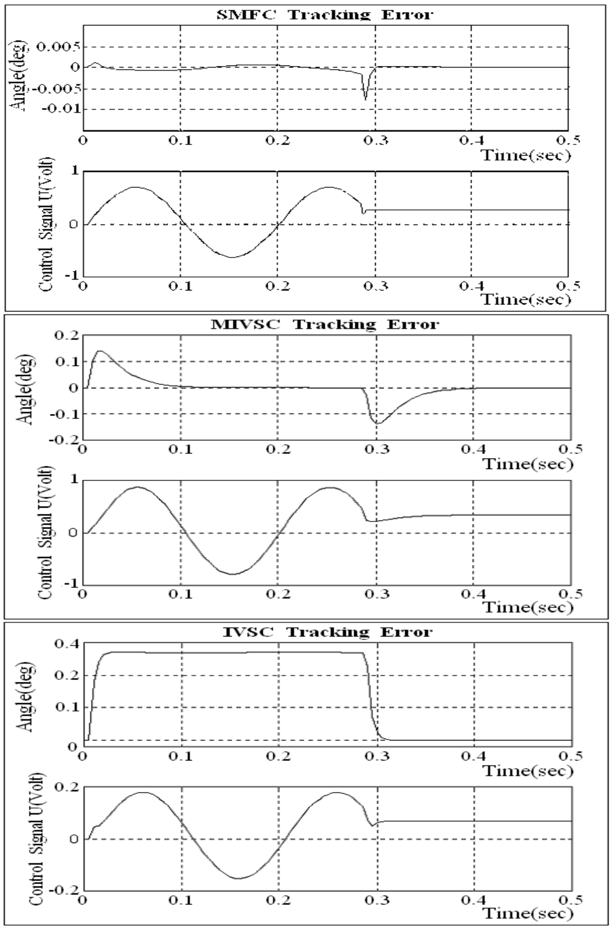

Fig. 4. Comparison of position tracking errors and control signal.

5. Simulation results and discussion

The simulation results of the dynamic responses(angle) are shown in Fig. 3 and Fig. 4, where a ramp command input is introduced. In addition, the electrohydraulic is applied with a shaft-angle-dependent external load disturbance T_L and variations of plant parameters K_v and J. The results are compared with those obtained from the IVSC and MIVSC approaches. These curves illustrate the robustness of the SMFC for electrohydraulic under various loads and abrupt disturbance. It is clear from the figures that SMFC approach can be maintained almost identically but vary significantly for other approaches. Fig. 4, shows the comparison of tracking errors and control signals under the same testing conditions. The smooth curve of the control function with the proper smoothing function clearly indicates that the smoothing function can eliminate chattering. From the observation, it is obvious that the proposed approach gives the minimum tracking error. That is, it gives a minimal tracking error and it also tracks the command input very closely during the change of the command input. Among them, the IVSC approach performs poorly. It gives a substantially sustained tracking error. Thus, the proposed approach seems amenable for practical implementation.

6. Conclusions

This paper described a position servo control systems for an electrohydraulic using a SMFC approach. Procedures are developed for choosing the control function for determining the coefficients of the switching plane and the integral control gain such that the system has desired properties. The application of SMFC to an electrohydraulic has show that the proposed approach can improved the tracking performance by 75% and 90% when compared to the MIVSC and IVSC approaches. Furthermore, the simulation results demonstrate that the proposed approach can achieve the requirements of robustness in the presence of plant parameter variation, load variations and nonlinear dynamic interactions. It is a robust and practical control law for electrohydraulic systems.

7. Acknowledgement

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