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การวิเคราะห์ตัวประกอบความเข้มของความเค้นในวัสดุต่างชนิดกัน ภายใต้ภาระทางความร้อน

Stress Intensity Factor in Dissimilar Materials under Thermal Loading

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บทคัดย่อ

ในรอยต่อของวัสดุต่างชนิดกันซึ่งมีผิวเชื่อมต่อภายในและอยู่ภายใต้ภาระการเปลี่ยนแปลงทางอุณหภูมินั้น การกระจายตัวของความเค้นรอบจุดมุมของรอยต่อทำการวิเคราะห์ได้ด้วยระเบียบวิธีทางบาวน์ดะรีเอลิเมนต์ และสามารถหาค่าตัวประกอบความเข้มของความเค้นที่เกิดขึ้นบริเวณรอบจุดมุมของรอยต่อสำหรับเทอมความเค้นอนันต์ในรูปของยกกำลังและในรูปของลอการิทมิกธรรมชาติ จากการเปลี่ยนแปลงคุณสมบัติของวัสดุที่นำมาต่อเชื่อมกันทำให้ทราบว่า ค่าตัวประกอบความเข้มของความเค้นสำหรับวัสดุต่างชนิดกันภายใต้ภาระการเปลี่ยนแปลงทางอุณหภูมิอย่างสม่ำเสมอนั้น เป็นสัดส่วนกับค่าการเปลี่ยนแปลงทางอุณหภูมิและแปรผันตามความแตกต่างของค่าสัมประสิทธิ์การขยายตัวเนื่องจากความร้อนของวัสดุต่างชนิดกัน

คำหลัก: ตัวประกอบความเข้มของความเค้น วัสดุต่างชนิดกัน ความเค้นจากความร้อน

Abstract

In a joint of dissimilar materials with an internal interface corner under thermal loading, the stress distributions around the vertex point can be investigated by BEM. The corresponding stress intensity factors for the stress singularities of power law and logarithmic law can be obtained. For various material combinations, it can be seen that the stress intensity factors in dissimilar materials under a uniform change in temperature are proportional to the temperature variation, and depend on the difference in the thermal expansion coefficients.

Keywords: Stress intensity, Dissimilar materials, Thermal stress.

1. Introduction

Dissimilar materials joints have been used in various engineering and structural applications. The bonded joints usually are composed of two-materials with different elastic properties and thermal expansion coefficients, such as solder joints, Brazing joints and adhesive bonded joint in electronic packaging applications. However, the unbounded stress possibly exists around the vertex of the bonded joint due to the discontinuity of material properties. Then, the failure of the joint and initial crack can occurs, and so that the function of the joint in the application could not maintain any longer. Therefore, it is interesting to investigate the characteristic of stress singularity fields around the vertex point in three-dimensional bonded joints under thermal loading for improving reliability of the bonded joint in engineering applications.

Due to the occurrence of stress singularity at the vertex point of dissimilar materials, failure occurs and the reliability of joints significantly decreases. It is well-know that the order of stress singularity, λ , is essential in the determination of the most serious state of stress around the singular point in the joint. The stress distribution around the singular point can be investigated using numerical methods (BEM or FEM). However, the order of stress singularity cannot be estimated directly from the stress distribution as a joint is made of materials yielding a complex number or several real numbers of the order of stress singularity. Therefore, the methods for directly investigating the order of stress singularity around the singular point are interesting discoveries. Furthermore, the stress intensity factors can be estimated together with every value of the order of stress singularity and the stress distribution using an interpolation function. Therefore, many investigations on the order of stress singularity have been carried out. For two-dimensional joints, Bogy reported that the order of stress singularity near the apex in a two-phase bonded structure depended on wedge angles and the Dundurs' parameters [1-4]. Dempsey and Sinclair reported that the order of stress singularity at the vertex of the N-material wedges also depended upon the number of materials, and logarithmic singularity can occurs [5]. Pageau et al. have demonstrated the order of stress singularity for all perfectly bonded two- and three-material junctions [6, 7]. Koguchi reported that the order of stress singularity near the apex in a three-phase bonded structure depended on four Dundurs' composite parameters, and can be reduced by a suitable arrangement of the bonded order of materials [8]. For three-dimensional joints, Benthem reported that the order of stress singularities for symmetrical and antimetrical states of stress at the vertex of a quarter infinite cracks in a half-space depended significantly on Poisson's ratio [9, 10]. Bazant and Estenssoro reported that the order of stress singularity at the vertex of inclined cracks meeting a half-space and the crack propagation depended on Poisson's ratio and crack front edge angle [11]. Pageau and Biggers reported that the order of stress singularities at the three-dimensional intersection of anisotropic multi-material junction with a free surface can be minimized by varying the fiber orientation [12, 13]. Koguchi reported that the order of stress singularities at the corner where four free surfaces and the interfaces of the three-dimensional joints meet depended on the Dundurs' composite parameters [14]. In the present study, the order of stress singularity for power-law singularity and the eigenvalues around the vertex point is investigated using the three-dimensional eigen value analysis by FEM. Then, the characteristics of the stress fields and the stress intensity factors around the vertex point under thermal loading are investigated by a three-dimensional BEM.

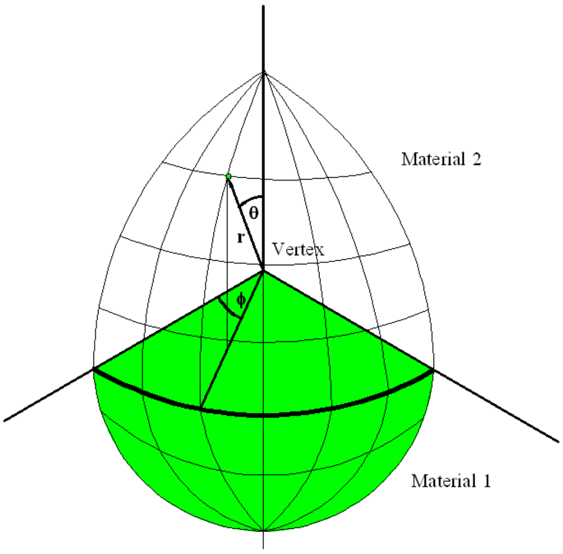

Figure 1. Model of a quarter sphere around the vertex point

2. The order of stress singularity

FEM formulation using an interpolation function of displacements, considering the stress singularity presented by Yamada and Okumura [16] and Pageau and Biggers [12, 13] is used to investigate the order of stress singularity around the vertex point. Figure 1 shows the model for analysis. The vertex point is at the origin in a spherical coordinate system. Taking the displacement at the origin as zero, the displacement in the *i* direction, u_i, at each node on the surface of FEM model near the vertex point can be expressed as follows:

$$u_i(r,\theta,\phi) = r^p f_i(\theta,\phi) \tag{1}$$

where $p=\lambda+1$. r,θ and ϕ are the spherical coordinates. λ is the order of stress singularity. Singular element with eight nodes using the serendipity quadratic interpolation function is employed for the present analysis. Angles θ and ϕ for spherical coordinates are expressed as

$$\theta = \sum_{j=1}^{8} H_j\theta_j \text{ , } \phi = \sum_{j=1}^{8} H_j\phi_j \tag{2}$$

where H_j represents the serendipity quadratic interpolation function. Then, the eigen equation is derived from the principles of virtual work for deducing the root *p*.

$$\left(p^2[A]+p[B]+[C]\right)\{u\}=0 \tag{3}$$

There are many roots, p, obtained from solving the eigen equation. Generally, the displacement fields around the singular point can be expressed as the following asymptotic series.

$$u_i = \sum_{a=1}^{n} r^{p_a} f_{ia}(\theta,\phi,p_a) \tag{4}$$

where $f_{ia}(\theta,\phi,p_a)$ is the angular variation of displacement fields in the *i* direction. p_a is the a-th root of an eigen equation. So, the eigen equation can be factorized for n roots as

$$\prod_{a=1}^{n}(p-p_a)=0 \tag{5}$$

when the multiple root of $p=p_1$ exist, Equation (5) is rewritten as

$$(p-p_1)^m \prod_{a=2}^{n}(p-p_a)=0 \tag{6}$$

where m is the number of the multiple root. In this case, the results of the displacement fields cannot be obtained by calculating only one term of factor, but necessarily correspond with all multiple terms of factor. The answer of *m*-th root can be obtained by calculating the derivative order $(m-1)$-th of the differential equation of the displacement fields with respect to p_1. Ordinarily, the differential equation of the displacement fields neglected the body force can be written by using the constitutive equations and strain-displacement relations as shown in the following expression.

$$N(u_i)=\left[\frac{\partial^2}{\partial x_j x_j}+\left(\frac{1}{1-2\nu}\right)\frac{\partial^2}{\partial x_i \partial x_j}\right]u_i=0 \tag{7}$$

where $N(\ )$ represents a differential operator, which also satisfies the following expression as

$$\frac{\partial^j}{\partial p_1^j}\left[N(u_i)\right] = N\left[\left(\frac{\partial^j u_i}{\partial p_1^j}\right)\right] = 0 \qquad (8)$$

If the displacement satisfies the differential equation and the *m*-th order term of factor exists, the displacements of the *m*-th order term of factor can be written as follows:

$$\begin{aligned} u_{i,m} &= \frac{\partial^{m-1}}{\partial p_1^{m-1}}\left[r^{p_1} f_{i1}(\theta,\phi,p_1)\right] \\ &= r^{p_1}(\ln r)^{m-1} g_{i1}(\theta,\phi,p_1) + r^{p_1}(\ln r)^{m-2} g_{i2}(\theta,\phi,p_1) \\ &+ \dots\dots + r^{p_1}(\ln r) g_{im-1}(\theta,\phi,p_1) + r^{p_1} g_{im}(\theta,\phi,p_1) \end{aligned} \qquad (9)$$

where g_{ib} (*b*=1,2..m) is the angular variation of the displacement fields in the *i* direction for the logarithmic singularity terms as $p = p_1$. For example, the displacement component of the fourth order term of factor, $u_{i,3}$, is written in the same way.

$$N(u_i) = N\left[\left(\frac{\partial u_i}{\partial p_1}\right)\right] = N\left[\left(\frac{\partial^2 u_i}{\partial p_1^2}\right)\right] = 0 \qquad (10)$$

$$\begin{aligned} u_{i,3} &= r^{p_1}(\ln r)^2 g_{i1}(\theta,\phi,p_1) + r^{p_1}(\ln r)^1 g_{i2}(\theta,\phi,p_1) \\ &+ r^{p_1} g_{i3}(\theta,\phi,p_1) \end{aligned} \qquad (11)$$

where if $p_1 = 1$,the displacement $u_{i,3}$ becomes as follow

$$\begin{aligned} u_{i,3}(r,\theta,\phi) &= r(\ln r)^2 g_{i1}(\theta,\phi) + r(\ln r)^1 g_{i2}(\theta,\phi) \\ &+ r g_{i3}(\theta,\phi) \end{aligned} \qquad (12)$$

Therefore, when three roots of $p_1 = 1$ exist, the displacement fields of the third order term of factor are composed of $r(\ln r)^2, r\ln r$ and r terms. Finally, the displacements fields around the singular point can be written by gathering the results for all terms of factor as follows:

$$\begin{aligned} u_i(r,\theta,\phi) &= r g_{i1}(\theta,\phi) + r(\ln r) g_{i2}(\theta,\phi) \\ &+ r(\ln r)^2 g_{i3}(\theta,\phi) + \sum_{a=2} r^{p_a} f_{ia}(\theta,\phi,p_a) \end{aligned} \qquad (13)$$

Then, the expressions for the stress fields are obtained through the differentiation of displacements.

$$\begin{aligned} \sigma_{ij}(r,\theta,\phi) &= L_{ij1}(\theta,\phi) + L_{ij2}(\theta,\phi)\ln r + L_{ij3}(\theta,\phi)(\ln r)^2 \\ &+ \sum_{a=2} r^{\lambda_a} K_{ija}(\theta,\phi,p_a) \end{aligned} \qquad (14)$$

where L_{ijm} is the stress intensity factor of the logarithmic singularity term (*m*=1,2,3), K_{ija} is that of the $r^{\lambda a}$ term (a=2,3…n), and $\lambda_a = p_a - 1$. The subscripts *i*, *j* refer r,θ and ϕ in a spherical coordinate system. The domain for the eigen value analysis by FEM at the vertex point is a quarter sphere which the free surface and the interface plane are taken at $\phi = 0, \pi/2$ and $\theta = \pi/2$, respectively as shown in figure 1. Mesh division with the square mesh size mapped on plane is employed as shown in figure 2. The number of elements for optimum calculation is 200 and the number of nodes is 661.

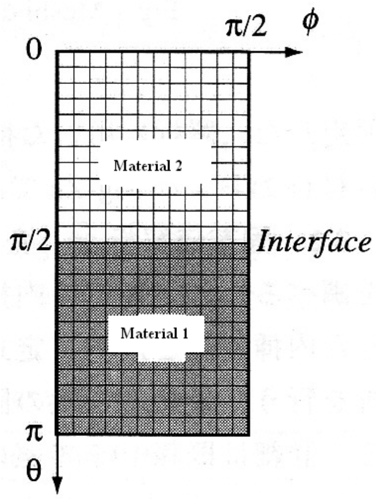

Figure 2. Mesh division for FEM eigen analysis

The stress fields at the vertex point with high stress are examined using BEM for thermoelasticity with a uniform temperature variation in dissimilar materials. α_{T1} and α_{T2} are the thermal expansion coefficients for material 1 and for material 2, respectively as shown in figure 3.

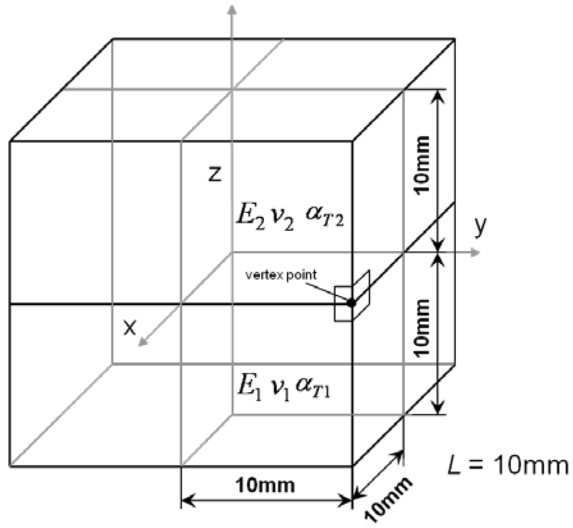

Figure 3. Model for BEM analysis

3. Results and Discussion

In the present study, material properties of Material 1 and Material 2 are defined as E_1 = 206 GPa, ν_1 = 0.3 and E_2 = 52.6742 GPa, ν_1 = 0.26316. It can be found that the multiple root of $(p=1)$ can be obtained using the three-dimensional eigen value analysis by FEM for the vertex point, while only the single root of $(p=1)$ can be obtained from the two-dimensional eigenvalue analysis by FEM for the apex in two-dimensional bonded joints as shown in Table 1. Therefore, from the theory mentioned at the beginning of the paper, the characteristic of stress singularity fields around the vertex point in the three-dimensional joints can be written possibly in a form of the combination of the r^{λ} term and the logarithmic singularity terms. For the three-dimensional FEM eigen value analysis, the stress fields can be expressed as follows:

$$\sigma_{ij}(r,\theta,\phi) = L_{ij1}(\theta,\phi) + L_{ij2}(\theta,\phi)\ln r + L_{ij3}(\theta,\phi)(\ln r)^2 + r^{\lambda}K_{ija}(p_a,\theta,\phi) \quad (15)$$

Table 1. Eigenvalues for $E_1 = 206GPa$, $\nu_1 = 0.30$, $E_2 = 52.6742GPa$ and $\nu_2 = 0.26316$

	Real part of p	Imaginary part of p	$\lambda = \mathrm{Re}(p) - 1$
3D			
1	0.8727359	0.0000000	-0.1272641
2	1.0000177	0.0000000	0
3	1.0000000	0.0000000	0
4	1.0000053	0.0000000	0
2D			
1	0.9071225	0.0000000	-0.0928775
2	1.0000000	0.0000000	0.0000000

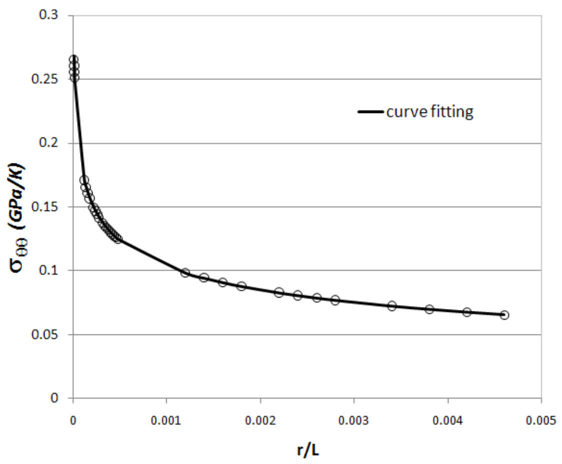

Figure 4. Curve fitting of $\sigma_{\theta\theta}$ at the interface for ΔT = -100 K

The order of stress singularity of a vertex in three-dimensional joints is obviously larger than the apex in two-dimensional joints in plane strain condition. Figure 4 shows the curve fitting of the stress distribution of $\sigma_{\theta\theta}$ at the interface for ΔT = -100K as α_{T1} =1.0x10^{-6}K^{-1} and α_{T2} =5.0x10^{-6}K^{-1}. The curve fitting performed by using equation (15) and λ =-0.1272641 is very good fit. Then, the stress intensity factors for ΔT = -100K and ΔT = -200K are investigated and show in Table 2. It can be seen that the stress intensity factors are proportional to the temperature change. The stress intensity factor $K_{\theta\theta 1}$ of the r^{λ} singularity term is larger than that for $L_{\theta\theta 2}$ and $L_{\theta\theta 3}$ of the logarithmic singularity terms. Therefore the power-law singularity term has more influence on the characteristic of stress fields around the vertex.

Figure 5 shows the effect of α_{T2} on stress intensity factor $(-K_{\theta\theta}/\Delta T)$ when α_{T1} is fixed to $1.0\text{x}10^{-6}\text{K}^{-1}$. It can be seen that the magnitude of the stress intensity factor increases linearly with α_{T2}.

Table 2. Stress intensity factor around the vertex

	$L_{\theta\theta 1}$	$L_{\theta\theta 2}$ (GPa/K)	$L_{\theta\theta 3}$	$K_{\theta\theta 1}$
3D ΔT= -100K				
	-0.0205	0.001497	0.001572	0.02454
3D ΔT= -200K				
	-0.0409	0.002996	0.003145	0.04902
2D ΔT= -100K				
	-0.1130	0.0	0.0	0.09026

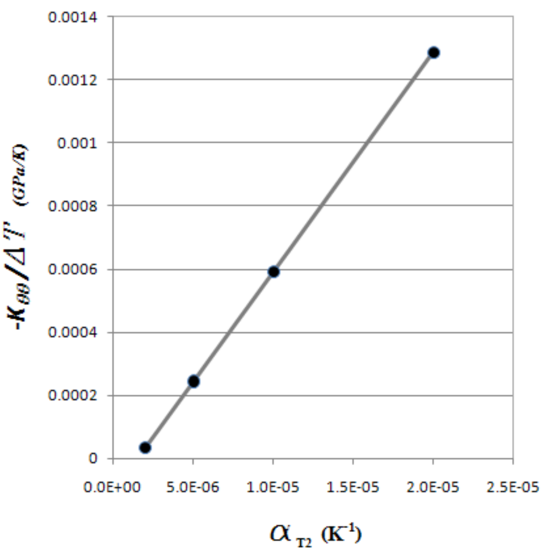

Figure 5. Effect of α_{T2} on $K_{\theta\theta}/\Delta T$

4. Conclusion

The eigenvalues including the order of stress singularity in a form of power-law singularity at the vertex point in three-dimensional joints were investigated using the FEM eigen analysis. The order of stress singularity at the vertex was larger than that at the apex in two-dimensional joints. The multiple root of $p=1$ existed. Then, the logarithmic singularity occurred at the vertex point in three-dimensional joints. The stress intensity factors of a power-law singularity term and logarithmic singularity terms have been investigated for the stress fields around the vertex point in three-dimensional joints under thermal loading obtained by BEM analysis. It can be concluded that the stress intensity factors were proportional to the temperature change, ΔT, and depended on the difference in the thermal expansion coefficients. Power-law singularity significantly influenced the characteristics of the stress fields around the vertex points under thermal loading.

5. Acknowledgement

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