The 17th Annual Conference of Mechanical Engineering Network of Thailand
15-17 October 2003, Prachinburi, Thailand

Buckling Load Characteristic of Conical Shells under Various End Conditions

Chawalit Thinvongpituk[1] and Hobb El-Sobky[2]
[1]Department of Mechanical Engineering, Faculty of Engineering, Ubon Ratchathani University
Warin Chamrab, Ubon Ratchathani, Thailand 34190
Tel: 045-288400-3 Ext. 3814-5 Fax: 045-288378 111 E-mail: enchaoth@ubu.ac.th
[2] Applied Mechanics Division, Mechanical, Aerospace and Manufacturing Department,
University of Manchester Institute of Science and Technology (UMIST), Manchester, UK, M60 1QD

Abstract

This paper is aimed to investigate the buckling load characteristic of conical shells subjected to axial loading. It was focused on the effect of end conditions, which are believed to promote the buckling resistance of structures. The study was carried out by means of experiment and structural model (FEA). The experimental result was, then, compared with the FEA and good agreement was achieved. The result suggested the strength of conical shells may be enhanced by constraining at the end. Particularly, the top constrained cones provide considerable high strength improvement (3%-10%).

1. Introduction

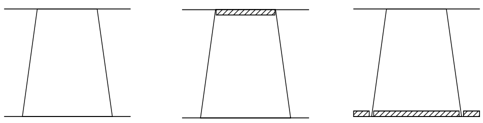
The study of thin shell structure has been given a considerable attention for many decades, aimed to improve criteria for the design of shell element. Among several shell structures, conical shell has gained high interest since it offers a unique energy absorption characteristic. The rate of energy absorbed by conical shell is increasing as the collapse progresses, while this rate is constant in case of cylindrical shell [1]. In addition, conical shell offers higher crush stroke compared to a cylindrical shell. Due to these advantages, the study of instability of cone has been one of the main issues in research of about thin shell. Seid [2, 3] may be a pioneer who presented a classical formula for buckling load of a cone under axial compression and under the influence of external hydrostatic. There were also other investigators such as Lackman and Penzien [4], Singer [5, 6], Weingarten et al [7], Lukasiewicz and Szyskowski [8] and Esslinger and Geier [9] who studied on the stability of conical shell using different approaches. Several formulas for buckling load of cone were proposed for different types of problem. It has been excepted that the end condition of a structure has great effect on its stability. In case of conical shells, there were some evidences from experiment and analytical results, such as Singer [10], Thurston [11], Tani et al [12], Petsios [13] and Pariatmono et al [14] suggested that the buckling load of end constrained cone is higher than the simply supported cone.

Nowadays, since the use of numerical models in studying shell behavior is widespread due to advance in computer technology. There are many FE packages that can be used effectively in this field of study, such as ABAQUS, INCA, PANDA and BOSOR. ABAQUS is a large-scale general-purpose computer code that performs the static, dynamic and heat transfer by finite element method. Schneider [15], Cryssanthopoulos [16], Psimolophitis [17] and Thinvongpituk [1] are some examples of investigator who used this package to investigate the behavior of shell structures.

This study is aimed to investigate the influence of end conditions on the buckling load characteristic of conical shell subjected to axial loading. The study used FE package (ABAQUS) and the result was verified by the experiment.

2. Experimental programme and the modeling

The experiment was carried out with ten specimens, which are truncated cones, made from aluminum, with top and bottom radii of 26mm and 77.56mm. The thickness of each specimen was varied from 0.4mm-0.95mm in order to obtain a range of mean radius to thickness ratio (R_m/t) from 60-143. This ratio was used to characterize the buckling behavior of specimens. Three different end conditions were used in the experimental programme i.e. simply supported ends, the top-constraint and the base-constraint end, as shown in Figure 1. It is well known that the conical shells usually start buckling from the smaller end by bending the edge inward. Therefore, the top-constraint was achieved by using step plate fits inside the top truncation, preventing the top edge from bending inward. While the bottom constraint was applied to the specimen by a grooved plate and let the bottom truncation sits in the groove. Consequently, there is no radial displacement at the bottom end.



A) B) C)

Figure 1 Illustrates the specimens with three end constraints: A) Simply Supported, B) Top-Constraint, C) Base-Constraint

The specimens were axially crushed, using a Universal Testing Machine, with a speed of 5mm/min. Load, displacement and buckling load were recorded.

The experiment was simulated using a FE package (ABAQUS). Each specimen was modeled with a number of shell elements type R4S5. The different end-constraints were prescribed by correctly defining any degree of freedom, which needs to be constrained. In addition, 17 more models were constructed in order to cover wider range of data and achieved finer results. Consequently, the FE models cover the range of R_m/t from 41-143 and with three different end conditions. In conclusion, a total of 27 models were used in this study. The dimension of the FE models and experimental specimens are shown in Table 1.

Table 1 Dimension of the experimental specimens and FE model

Mean Radius to Thickness Ratio (R_m/t)	Thickness t (mm)	Height H (mm)
41	1.4	106.0
52	1.1	106.0
60	0.95	106.6
67.5	0.85	106.9
72	0.8	106.2
88	0.65	102.8
96	0.6	104.2
114	0.5	105.2
143	0.4	103.5

3. Results and Discussion

The buckling load of specimens, achieved from experiment and FEA are listed in Table 2. In general, the buckling loads predicted from FE models are fairly close to the experimental results. A small discrepancy between them may be attributed to non-uniformity of specimen thickness and friction of the contact surface between specimen and testing machine.

Table 2 The buckling loads of tested specimens and FE models

Mean Radius to Thickness Ratio (R_m/t)	Buckling Load (kN)					
	Simply Supported		Base Constrained		Top Constrained	
	Test	FEA	Test	FEA	Test	FEA
41	-	15.19	-	15.21	-	16.65
52	-	11.11	-	11.14	-	11.9
60	8.51	9.19	-	9.2	-	9.7
67.5	8.39	7.94	-	7.95	8.27	8.31
72	-	7.32	8.42	7.35	-	7.64
88	5.15	5.37	-	5.39	-	5.58
96	-	4.94	6.62	4.95	5.19	5.10
114	3.52	3.81	-	3.84	-	4.0
143	-	2.827	2.39	2.83	3.4	2.95

From Table 2, it can be seen that the conical shells with partially constrained end, either top or bottom end, offer higher buckling load than the simply supported cones. It could be said that constraining the end may enhance the strength of conical shell structure. This may be because constraining the edge reduces the radial displacement (*w*), which is believed to cause lower buckling load in the shell structure [2, 6, 10, 18].

The increment of buckling load due to end constraint is summarized in Table 3. It is very significant for the top constrained cone, that the buckling load of specimen is increased from 3%-10%. For the moderate and high values of R_m/t (60-143), the buckling load increases about 3%-5%. At low values of R_m/t (41-52), the top constrained cone offers about 7% and 10% increment of buckling load, compared to the simple support end. The effect of the base constraint to the buckling load of specimen is found to be small. The buckling load increases not more than 1% for the base constrained cone. A possible explanation of this is that the conical shell generally starts buckling from the top edge where the influence of base constraint is least.

Tables 3 Increment of the buckling load due to the end constraints

Mean Radius to Thickness Ratio (R_m/t)	Incremental Buckling Load (%)	
	Base Constrained	Top Constrained
41	0.14	9.62
52	0.3	6.98
60	0.09	5.4
67.5	0.13	4.7
72	0.39	4.4
88	0.33	3.9
96	0.1	3.1
114	0.89	4.9
143	0.1	3.9

The buckling loads were also plotted against the mean radius to thickness ratio (R_m/t) as shown in Figure 2 (A) for the simply supported specimens, (B) for the Top-Constraint and (C) for the Base-Constraint. It is observed that the buckling load of conical shell decreases exponentially from lower R_m/t to higher R_m/t. The changing gradient of buckling load line is relatively high at low value of R_m/t and becomes smaller when R_m/t is getting higher. These are in the same pattern, no matter what the end conditions are.

It is worth mention that there are some reports about the buckling mode of conical shells that they generally buckle by either expanding the bottom edge radially outward or bending the top edge inward [13, 17, 19]. These can be classified in to different modes of collapse and can be linked to a parameter R_m/t and types of end constraint. Detail of this issue may be obtained from [1, 17].

4. Conclusion

The buckling characteristic of truncated aluminum cones has been investigated in this paper. The study programme included experiment and FEA using a commercial FE package (ABAQUS). The result achieved from FE model agreed well with the experimental result. The buckling load of conical shells was characterized with the parameter R_m/t. It was found that the buckling load of cone decreases exponentially from lower R_m/t to higher R_m/t. The result also suggested that the buckling resistance of conical shell may be increased by constraining at the edge. Particularly, the conical shell with top-constraint provides about 3%-10% higher buckling load than the simply supported cones. It was also observed that the top constraint has much higher influence on the stability of cone than the base constraint. The evidence of this is the increment of buckling load for the base constrained end is only less than 1% while it is almost 10% in case of top constrained end.

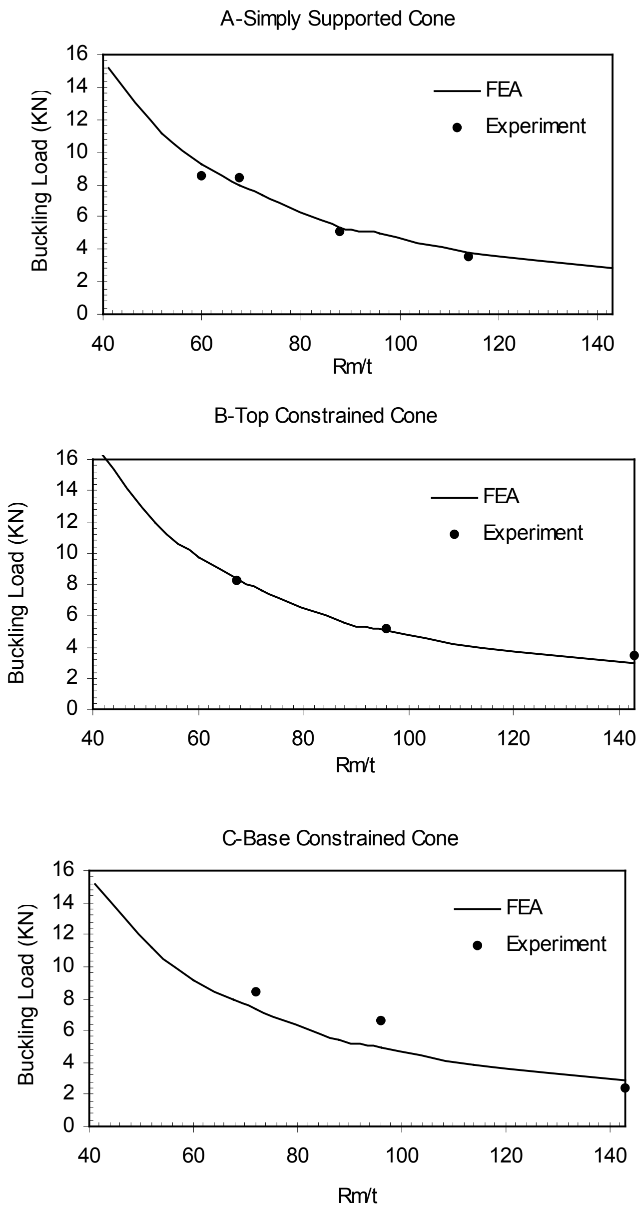


Figure 2 Buckling load characteristic of conical shells with various end conditions.

References

[1] Thinvongpituk C and El-Sobky H, “The Effect of End Conditions on the Buckling Load Characteristic of Conical Shells Subjected to Axial Loading” Proceeding of the ABAQUS Users’ Conference 2003, 3-5 June 2003, Munich, Germany

[2] Seide P. "Axisymmetrical Buckling of Circular Cones under Axial Compression" Trans. ASME, Journal of Applied Mechanics, 1956 pp.625-628

[3] Seide P. "Calculations for the Stability of Thin Conical Frustums Subjected to External Uniform Hydrostatic Pressure and Axial Load." Journal of Aerospace Science, vol. 29(8), 1962, pp.951-955

[4] Lackman L., Penzien J. "Buckling of Circular Cones under Axial Compression." Trans. ASME, 1960, pp.458-460

[5] Singer J. "Buckling of Circular Conical Shells under Axisymmetrical External Pressure". Journal of Mechanical Engineering Science, vol. 3(4), 1961, pp.330-339

[6] Singer J. "Buckling of Circular Conical Shells under Uniform Axial Compression." AIAA, vol. 3(5), May 1965, pp.985-987

[7] Weingarten V.I., Morgan E.J., Seid P. "Elastic Stability of Thin-Walled Cylindrical and Conical under Combined Internal Pressure and Axial Compression." AIAA, vol. 3(6), 1965, pp.1118-1125

[8] Lukasiewicz S. and Szyszkowski, W. "Geometrical Analysis of Large Elastic Deflections of Axially Compressed Cylindrical and Conical Shells." International Journal of Non-linear Mechanic, vol. 14, 1979, pp.273-284

[9] Esslinger M and Geier B. "Buckling and Postbuckling Behaviour of Conical Shells Subjected to Axisymmetric Loading and of Cylinders Bubjected to Bending." Theory of Shells, 1980, pp.263-288 (Edited by Koiter, W.T. and Mikhalov, BK.) North Holland Publishing Comp. Amsterdam

[10] Singer J. "The effect of Axial Constraint on the Instability of Thin conical shells under external pressure." Trans. ASME, Journal of applied Mechanics, March 1962, pp.212-214

[11] Thurston G.A. "Effect of Boundary Conditions on the Buckling of Conical Shells under Hydrostatic Pressure." ASME, Journal of Applied Mechanics, 1965, pp208-209

[12] Tani J. and Yamaki N. "Buckling of Truncated Conical Shells under Axial Compression". AIAA Journals, vol. 8(3), 1970, pp.568-571

[13] Petsios, M. "Buckling of Thin Truncated Conical Shells (Frusta) under Quasi-Static and Dynamics Axial Load" MSc. Dissertation, 1993, University of Manchester Institute of Science and Technology (UMIST), Manchester, UK

[14] Pariatmono N., Chryssanthopoulos M.K. "Asymmetric Elastic Buckling of Axially Impressed Conical Shells with Various end Conditions." AIAA Journals, vol. 33(11), 1995, pp.2218-2227

[15] Schneider M.H. "Investigation of the Stability of Imperfection Cylinders Using Structural Model." Engineering and Structures, vol. 18(10), 1996, pp.792-800

[16] Chryssanthopoulos M.K. and Spagnols A. "The Influence of Radial Edge Constraint on the Stability of Stiffened Conical Shells in Compression" Thin-walled structures, vol. 27(2), 1997, pp.147-163

[17] Psimolophitis, S.E. "An Investigation of Axisymmetric Shells under Axial Loading" Ph.D. Thesis 1997, University of Manchester Institute of Science and Technology (UMIST), Manchester, UK

[18] Donnell L.H and Wan C.C. "Effect of Imperfection on Buckling of Thin Cylinder and Columns under Axial Compression." Trans ASME, Journal of Applied Mechanics, March 1950, pp.73-83

[19] El-Sobky H., Singace A.A. and Petsios M. "Mode of Collapse and Energy Absorption Characteristics of Constrained Frusta Under Axial Impact Loading" International Journal of Mechanical Science, vol. 43, 2000, pp.743-757