Buckling of Axially Compressed Conical Shells of Linearly Variable Thickness Using Structural Model

Chawalit Thinvongpituk[1] and Pisit Techarungpaisarn[2]

[1]Department of Mechanical Engineering, Faculty of Engineering, Ubon Ratchathani University
Warin Chamrab, Ubon Ratchathani, Thailand 34190
Tel: 045-288400-3 Ext. 3814-5 Fax: 045-288378, E-mail: enchaoth@ubu.ac.th

[2]Department of Mechanical Engineering, Faculty of Engineering, Ubon Ratchathani University
Warin Chamrab, Ubon Ratchathani, Thailand 34190
Tel: 045-288400-3 Ext. 3814-5 Fax: 045-288378, E-mail: enchaoth@ubu.ac.th

Abstract

The present study is mainly concerned with the effect of thickness variation on the stability of conical shells under axial compression. The study was conducted with a series of experiment and structural model, using FE package (ABAQUS). The experiment was carried out with a number of specimens that were fabricated with constant thickness. Each specimen was crushed until the buckling point and buckling load was recorded. The computer model of each specimen was also constructed and used for simulating the experiment. The results achieved from the FE model were agreed well with the experimental results. The results of this study revealed that the buckling resistance of conical shells with variable thickness is reduced as the thickness reduction parameter increases. The losses of buckling resistance due to the variation of thickness may be approximated by a quadratic equation. This equation can be reduced to a linear equation when higher order is omitted. This expression is in the same form as Koiter's equation for a cylinder of linearly variable thickness [1]. It may be concluded that the reduction of the buckling resistance of cylindrical and conical shells due to their small variable thickness is proportional to the thickness reduction parameter and can be expressed in a simple linear equation.

1. Introduction

The study of thin shell structures has been given considerable attention for at least 6 decades, especially during wartime because of their importance in aircraft and missile applications. Various shapes of shell have been investigated such as hemispherical, conical and cylindrical shells. Research in shell structures has been approached from different angles, resulting the advances in theory and applications. Although the behavior of shells has been studied extensively for several decades, the influence of thickness variation on its' stability has not gain sufficiently attention and remains mysterious. In general, the thickness of shells is usually assumed to be uniform and constant in order to simplify the problems. In fact, the thickness is rarely constant. Moreover, in many applications, it is necessary to use non-constant thickness shell, for example joining pipes of unequal thickness using a variable conical pipe. It has been only few investigators who have carried out the study on the behavior of thin shell with variable thickness. Timoshenko [2] may be the first who investigate a simply problem of cylindrical tank with non-uniform thickness. Reissner [3] presented a mathematical analysis on the stability of conical shell of variable thickness, but the equation was too complicated and only a simple case was solved. Flugge [4] did similar analysis as Reissner [3] but with simpler method. De Silva and Naghdi [5] provided asymptotic solutions for simple shell of revolution with variable thickness. Bushnell and Hoff [6] published an investigation on the influence of parabolic variable thickness on the stability of cylinder. Recently, Gusic et al [7] used commercial FE packages (INCA and ABAQUS) to investigate the buckling characteristic of cylinder with harmonic thickness variation. The buckling of non-uniform thickness composite cylinder was studied extensively by Li et al [8, 9]. One of the distinctive reports was presented by Koiter [1]. From his mathematical analysis, he concluded that the buckling load of cylinder is reduced as the degree of thickness variation is increasing and it can be written in a quadratic form.

This study is aimed to investigate the effect of thickness variation on the stability of cone subjected to axial loading. The study programme is included the experiment and FE model.

2. Study Plan

2.1 Experimental Procedure, Specimens and FE Model for constant thickness conical shells

The experiment was carried out with 5 truncated aluminum conical shells. These specimens have semi-vertex angles of 18.5º, 20.5º and 25.5º. Each conical shell has constant thickness and covers the value of mean radius to mean thickness ratio (R_m/t_m) from 61-114. The specimens were tested with an Instron Testing System under quasi-static axial compression and under simply supported end only. They were crushed until the first buckling point and the buckling load was recorded. In order to compare, the FE model was constructed using a commercial FE package (ABAQUS). This was to simulate the experiment and the result achieved from the model was verified by the experimental result before use this code to investigate further cases.

In order to cover wider range of data, 5 more FE model were constructed to have values of R_m/t_m of 39, 48, 94, 121 and 141. The geometry of tested specimens and additional FE models is shown in Table 1.

Table 1 Geometry of tested specimens and additional FE model

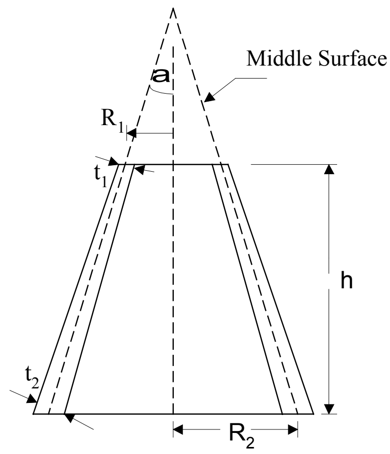
Cone No.	R_m/t_m	Thickness t (mm)	Top Radius (R_1)	Base Radius (R_2)	Semi-Vertex Angle (a)
1[1]	39	1.34	28.3	77.2	17.5
2[1]	48	1.15	26.85	77.7	18.5
3[2]	50	1.1	26.15	77.7	18.5
4[2]	61	0.95	26.85	77.5	25.5
5[2]	78	0.7	26	76.1	20.5
6[2]	87	0.65	26.4	75.45	25.5
7[1]	94	0.6	26.2	75.9	25.5
8[2]	114	0.5	26.5	76.7	25.5
9[1]	121	0.45	25.95	75.7	20.5
10[1]	141	0.4	26.15	75.5	25.5

[1] FE models to cover wider range of data

[2] Experimental specimens

2.2 Geometry of Variable Thickness Conical Shells

In order to investigate the influence of linearly variable thickness, a number of FE models was constructed to have thickness changed from top to bottom truncation. The procedure to construct these FE models may be found from [10]. Figure 1 illustrates the geometry of a model of non-constant thickness cone. The top radius (R_1) and bottom radius (R_2) are measured to the middle surface of structure. The wall thickness (t)



increases linearly from

Figure 1 Geometry of linearly variable thickness conical shell

minimum thickness (t_1) at top truncation to the maximum thickness (t_2) at the bottom. The increment of thickness follows equation (1), as below:

$$t = t_1(1 + x\varepsilon) \quad (1)$$

Where

ε is a thickness ratio parameter and defined as equation (2)

$$\varepsilon = \frac{(t_2 - t_1)}{t_1} \quad (2)$$

Where

t_1 and t_2 are minimum and maximum wall thickness of shell

x is the slant distance parameter from the top truncation along the surface of cone and defined as $x = \frac{x_2 - x_1}{L_s}$

Where X_1, X_2 are the slant distances, measured from apex of the cone to top and bottom truncations respectively. L_S is the slant length of the truncated cone, see Figure 2.

A number of FE models was built with five different values to thickness ratio parameter, which are ε=0 (constant thickness cone), ε=0.1, 0.5, 1.0 and ε=1.5. These were achieved by varying the minimum and maximum thickness (t_1 and t_2) of shell wall. However, the mean thickness (t_m) of non-constant thickness cone was kept equal to the average thickness of constant thickness specimens.

These structural models of truncated cone were constructed with a number of shell elements, type S4R5 using FE package (ABAQUS). The models, then, were crushed with two rigid elements, in the same manner as the experimental programme. The buckling load of each model was, then, recorded.

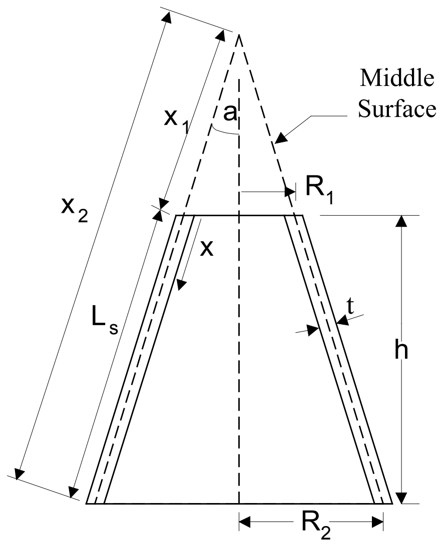


Figure 2 Geometry of normal cone, illustrates some geometric parameters (X, X_1, X_2, L_S, t)

3. Results and Discussion

The buckling load of constant and non-constant thickness conical shells, achieved from FE model and experiment, are presented and compared in Table 2.

Table 2 Buckling load of constant and non-constant thickness cones

R_m/t_m	Buckling Load (P_{cr}, kN)					
	Constant Thickness (ε=0)		Non-Constant Thickness[1] ($\varepsilon \neq 0$)			
	Test	FEA	ε=0.1	ε=0.5	ε=1.0	ε=1.5
39	-	18.6	17.57	13	7.96	3.7
48	-	13.9	13.34	9.9	6.34	3.1
50	13.45	13.1	12.45	9.27	5.9	2.9
61	8.51	7.9	7.47	5.73	3.7	2
78	7.34	6.6	6.2	4.62	3	1.5
87	5.15	4.74	4.5	3.42	2.22	1.21
94	-	4.3	4.0	3.1	2.02	1.1
114	3.52	3.53	3.3	2.56	1.7	0.96
121	-	3.64	3.41	2.57	1.6	0.86
141	-	2.51	2.37	1.81	1.2	0.67

[1] Results are from FEA only

From Table 2, it is observed that the buckling load of constant and non-constant thickness cones decreases as the mean radius to mean thickness ratio (R_m/t_m) increases. For the constant thickness cone (ε=0), the buckling load predicted from FE model is fairly close to the experimental value. The difference between experiment and prediction is less than 10%. The discrepancy between them may be attributed to non-uniformity of specimen thickness and friction of the contact surface between specimen and testing machine.

It is also observed that, the buckling load of structure decreases as the thickness ratio parameter (ε) is increasing. In order to characterize the influence of thickness variation on the stability of non-constant thickness cone, another parameter called buckling load reduction factor (ζ) is introduced. It is defined as a ratio of the buckling load of non-constant thickness cones to the buckling load of constant thickness cone.

$$\zeta = \frac{P_{cr}}{P_{cr,const}} \tag{3}$$

Where

P_{cr} is the buckling load of non-constant thickness cone

$P_{cr,const}$ is the buckling load of constant thickness cone

The buckling load reduction factors (ζ) of non-constant thickness cone with different thickness ratio parameter (ε) are shown in Table 3.

Table 3 Buckling load reduction factor (ζ) of non-constant thickness cone

R_m/t_m	Buckling Load Reduction (ζ)				
	ε=0.0	ε=0.1	ε=0.5	ε=1.0	ε=1.5
39	1	0.94	0.7	0.43	0.2
48	1	0.95	0.71	0.45	0.22
50	1	0.95	0.71	0.45	0.22
61	1	0.95	0.72	0.47	0.25
78	1	0.94	0.7	0.45	0.23
87	1	0.95	0.72	0.47	0.25
94	1	0.94	0.72	0.47	0.26
114	1	0.93	0.72	0.47	0.27
121	1	0.94	0.7	0.45	0.24
141	1	0.94	0.72	0.47	0.27

Surprisingly, it is observed from Table 3 that the buckling load reduction values (ζ) of cones are almost constant for specimens with the same thickness ratio parameter (ε). At the thickness ratio parameter (ε) of 0.1, the buckling load of cone reduces about 5%. The cones lose its' stability by 30% at thickness ratio of 0.5. The stability of cone is reduced by half for the value of ε=1 and it is reduced as much as 75% when the thickness ratio reaches 1.5.

Since the buckling loads reduction (ζ) of specimens with the same value of ε are almost constant, therefore, they may be approximated by their average as shown in Table 4.

Table 4 Average value of buckling load reduction factors for cones of different thickness ratio parameters

Thickness Ratio Parameter (ε)	Averaged Value of Buckling Load Reduction (ζ)
0	1
0.1	0.943
0.5	0.712
1.0	0.458
1.5	0.241

The averaged buckling load reduction factors in Table 4 are then plotted against the thickness ratio parameter, as shown in Figure 3.

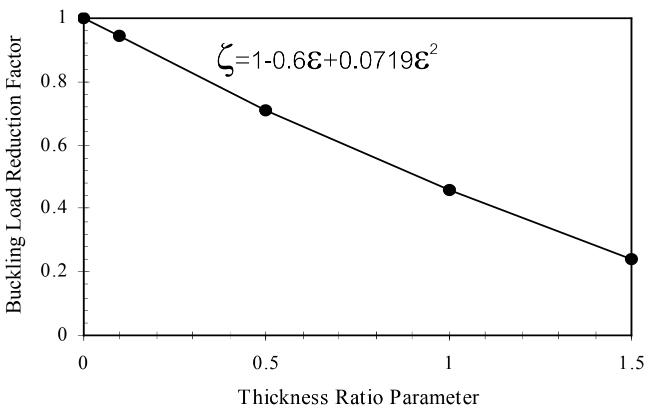


Figure 3 Relationship of buckling load reduction factor (ζ) and thickness ratio parameter (ε).

Figure 3 illustrates the relationship of the buckling load reduction of truncated conical shells due to thickness variation. It is obviously seen that the stability of cone is reducing as the thickness ratio parameter increases. By fitting curve, the relation of buckling load reduction factor (ζ) and thickness ratio parameter (ε) can be approximated by an expression below;

$$\zeta = 1-0.6\varepsilon+0.0719\varepsilon^2 \qquad (4)$$

If the thickness variation parameter (ε) is very small, equation (4) can be reduced to

$$\zeta = 1-0.6\varepsilon \qquad (5)$$

Equation (5) is in the same form as the Koiter's formula [1], which was proposed for the buckling load reduction of cylindrical shell due to thickness variation in axial direction. Koiter's equation is written as;

$$\zeta = 1-\varepsilon \qquad (6)$$

The difference between equation (5) and equation (6) is the constant value before ε. From equations (5) and (6), they may be suggesting that the buckling load reduction of cylindrical and conical shells with non-constant thickness can be linearly related to their thickness variation parameter, and can be summarized in one equation as;

$$\zeta = 1-K\varepsilon \qquad (7)$$

Where K=1 for cylindrical shell and K=0.6 for aluminum truncated conical shells.

4. Conclusion

This paper has investigated the effect of thickness variation on the stability of truncated conical shells under simply supported end and subjected to axial compression. It is aimed to emphasize the influence of thickness variation on the stability of conical shells. The study began with the investigation of stability of constant thickness cone, using FE model and experiment. The buckling load obtained from experiment and FE model was compared and good agreement was achieved. The FE model, then, used for further investigation on the stability of non-constant thickness cone. It was found that the variation of thickness in axial direction results in the reduction of buckling load. The buckling load reduction factor (ζ) can be related to the thickness variation parameter (ε) as expressed in equation (5) for small value of ε. This is in the same form as Koiter's equation [1] for cylinder of axially variable thickness. Equation (7) is a suggested expression to predict the buckling load reduction for, both, conical and cylindrical shells having thickness variable in axial direction but use different values of K. It may be concluded that the reduction of buckling load of cylinder and cone, due to their small thickness variation in axial direction, is proportional to the thickness reduction parameter.

It should be noted here that this study was carried out under simply supported only. Further investigation should be involved with other end conditions and more experimental work is suggested.

References

[1] Koiter W.T. et al "Buckling of an axially compressed cylindrical shell of variable thickness" Int. J. Solid Structures, 31(6), 1994, pp.797-805

[2] Timoshenko S "Theory of plates and shell" McGraw Hill, New York, 1940

[3] Reissner E. "On the theory of thin elastic shells" H Reissner Anniv., 1949, pp.231-247

[4] Flugge W, "Stress in shells" 2nd ed., 1973, Springer-Verlage. Berlin

[5] De Silva, C.N. and Naghdi, P.M., "Asymptotic solution of a class of elastic shells of revolution with variable thickness" Quart. Appl. Math., 15(2), 1957, pp.169-182

[6] Bushnell, D. and Hoff, N .J. "Influence of a circular cylindrical shell with rapidly varying parabolic wall thickness" AIAA, 2(12), 1964, pp.2167-2173

[7] Gusic G., Combescure A., Jullien J.F. "The influence of circumferential thickness variations on the buckling of cylindrical shell under external pressure" Computer & Structures, 74, 2000 pp.461-447

[8] Li, Y. W., Elishakoff, I. and Starnes, J.H. "Axial buckling of composite cylindrical shells with periodic thickness variation" Computers & Structures, 56(1), 1995, pp.65-74

[9] Li, Y. W. et al "Effect of the thickness variation and initial imperfection on buckling of composite cylindrical shells: asymptotic analysis and numerical results by
BORSOR4 and PANDA2" Int. J. Solids Structures, 34(28), 1997, pp.3755-3767

[10] Thinvongpituk C. "The Behaviour of Non-Constant Thickness Conical Shells under Axial Loading" Ph.D. Thesis, 2002, University of Manchester Institute of Science and Technology (UMIST), Manchester, U.K.