The 17th Conference of Mechanical Engineering NETwork of Thailand
15-17 October 2003 Prachin Buri

Parametric Study of the Acoustic Transmission Loss of Multiple Helmholtz Resonator-Type Silencers

Ming Lokitsangtong Shuntaro Murakami
Department of Mechanical Engineering, Faculty of Engineering, King Mongkut's Institute of Technology Ladkrabang
3 Moo 2 Chalongkrung Road, Ladkrabang, Bangkok 10520, Thailand
Tel: 0-23264197 Ext. 107 Fax: 0-23264198 E-mail: klming@kmitl.ac.th

Abstract

Plane acoustic waves have many characteristics in common with the longitudinal waves that are propagated along a thin bar. A vibrating bar may be loaded with some kind of mechanical impedance for instance a concentrated mass. The transmission of compressional waves along a bar is much dependent on the ratio of the concentrated mass and the bar mass. In line with this consideration the characteristic impedance of a multiple Helmholtz resonator-type silencer can be treated by proportioning out the plane wave front propagating through the flow duct. In this study, silencers with identical-size resonators which their transverse components are symmetrically arranged have been investigated. In the present paper the concept based on that of dividing the overall cross-sectional area of the flow duct into equal cross-sectional areas corresponding to individual resonator has been implemented on a comprehensive computer program to evaluate the acoustic performance of silencers with multiple Helmholtz resonators. A parametric study is conducted for acoustic properties such as transmission loss, resonance frequency, characteristic impedance etc., mechanical properties such as Mach number etc., and geometric properties such as volume of the resonance chamber , connector diameter and its length etc.

1. Introduction

Helmholtz resonators have been used for several decades for radiation of sound so as to reduce noise from exhaust and ventilation systems. On the basis of the linear wave theory, disregarding flow, Davis et al, presented the equations for a single resonator and for the multiple resonators[1]. These have contributed to the reduction of narrow frequency-band noise traveling in a duct with fairly low-speed flow. The additional investigations have indicated that the performance of a single resonator drops as flow speed is increased [2,3]. The reason appears to be induced by entropy fluctuations at discontinuity [4]. Since such energy dissipation may occur close to the resonator entrance and are almost unable to be controlled, the multieffect of resonators is expected to obtain the noise reduction required.

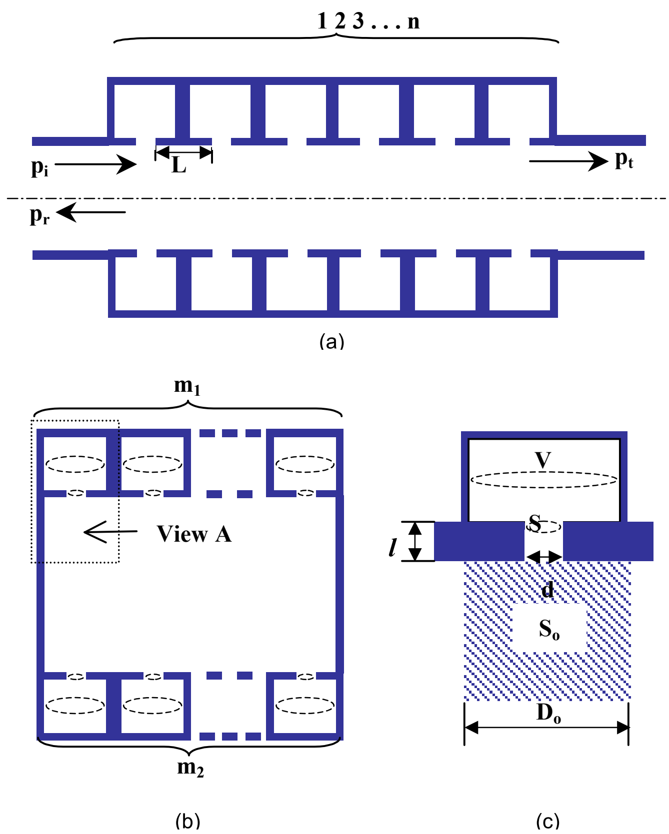

Fig.1 Silencer model (a) longitudinal (b) transverse (c) A, sections

Plane waves of sound propagated through a duct are governed by the linear wave equation. In the case of a thin bar, equation of motion describes longitudinal waves of vibration propagated in it. Naturally both of these equations are of the same form. In Helmholtz resonator the outer opening of the connector radiates sound, providing radiation resistance and a radiation mass. The fluid in the connector, moving as a unit, provides another mass element. The compression of the fluid in the chamber provides stiffness[5]. A vibrating bar may be loaded with some kind of mechanical impedance for instance a concentrated mass. The transmission of compression waves along a bar is much dependent on the ratio of the concentrated mass and the mass of the bar. In line with this consideration the characteristic impedance of a multiple Helmholtz resonator-type silencer can be treated by proportioning out the plane wave front propagating through the flow duct. In this paper some general design guidelines based on the parametric studies have been arrived for silencers with congruent Helmholtz resonators which their transverse components are symmetrically arranged and contiguous to each other.

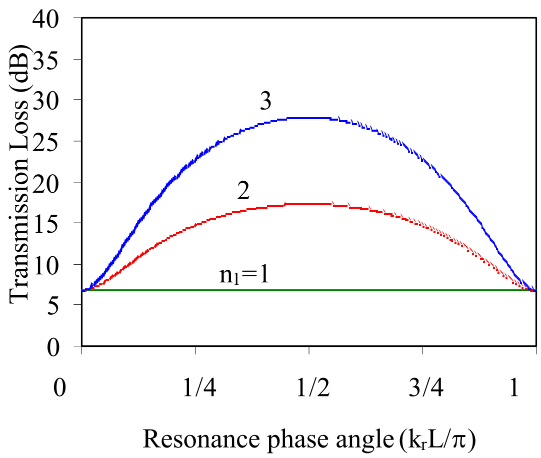

Fig.2 Relationship between the transmission loss and phase angle at resonance frequency.

2. Equations

A silencer composed of multiple Helmholtz resonators, which are arranged on either of the opposite sides of a rectangular duct terminating with the anechoic end, is depicted in Fig.1(a) (vertical section) and Fig.1(b) (cross section), where m_1, m_2 denote the number of transverse resonators, n the number of total resonators in the longitudinal direction, and L connecting length of a duct between any two contiguous resonators. Fig.1(c) shows the detail of one resonator as shown in view A in Fig.1(b), where symbols are explained as follows ; V: volume of resonance chamber, *l*: actual length of connector, l_e: effective length of connector, d: diameter of connector, S: cross-sectional area of connector, and Do, So: width and cross-sectional area of a partial duct corresponding to individual resonator, respectively.

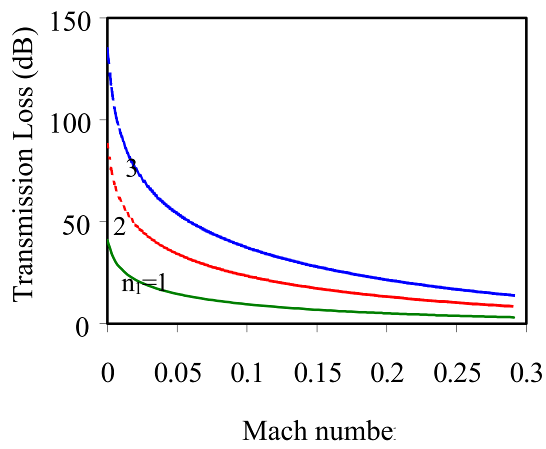

Fig.3 Relationship between the transmission loss and Mach number at resonance frequency

For an expansion chamber type silencer, Alfredson and Davies presented the theoretical treatment introducing the term of energy dissipation by entropy fluctuations into the momentum, energy, and continuity equations set up at a discontinuity[6]. On the basis of this concept, Munjal derived a transfer matrix in connection with sound pressure and mass rate at the duct sections directly at the front and back of a resonator[7]. If volume velocity is used instead of mass rate in the above matrix, matrix elements A,B,C,D may be rewritten as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{2M + \frac{Z_r}{Z_0}} \begin{bmatrix} M + \frac{Z_r}{Z_0} & M^2 Z_0 \\ \frac{1}{Z_0} & M + \frac{Z_r}{Z_0} \end{bmatrix} \quad (1)$$

In eq. (1), M is Mach number of mean flow passing over the resonator and Z_o is the characteristic impedance of the duct given by

$$Z_0 = \frac{\rho_0 c}{S_0} \quad (2)$$

where ρ_o is mean density of medium, c is sound speed, and additionally Z_r is the acoustic impedance of a resonator given by

$$Z_r = R + j(X + R) \quad (3)$$

where R is connector resistance and X is resonator reactance with the resistance term omitted. Therefore the dimensionless impedance Z_r/Z_o may be expressed by

$$\frac{R}{Z_0} = \frac{16}{\left(\frac{d}{\ell_e}\right)\left(\frac{d}{D_0}\right)^2}\sqrt{\frac{\mu f}{\pi\rho_0 c^2}} \quad (4)$$

and

$$\frac{X}{Z_0} = S_0\left(\frac{f}{f_r} - \frac{f_r}{f}\right)\sqrt{\frac{\ell_e}{VS}} \quad (5)$$

where μ denotes dynamic viscosity of medium, f frequency, and f_r resonance frequency given by

$$f_r = \frac{c}{2\pi}\sqrt{\frac{S}{V\ell_e}}$$

The transfer matrix for a duct of connecting length L between two neighboring resonators can be obtained from the analysis in ref. [4], that is,

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = e^{-jkLM}\begin{bmatrix} \cos kL & jZ_0 \sin kL \\ \frac{j\sin kL}{Z_0} & \cos kL \end{bmatrix} \quad (6)$$

and

$$k = \frac{2\pi f}{c\left(1 - M^2\right)} \quad (7)$$

Thus sound pressure p_I and volume velocity q_I at the silencer entrance may be related to both quantities p_{II} and q_{II} at its exit, through a series of matrices which can be given as

$$\begin{pmatrix} p_I \\ q_I \end{pmatrix} = \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix}\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix}\begin{pmatrix} A_2 & B_2 \\ C_2 & D_2 \end{pmatrix}\begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix}\cdots$$
$$\cdots\begin{pmatrix} a_{n-1} & b_{n-1} \\ c_{n-1} & d_{n-1} \end{pmatrix}\begin{pmatrix} A_n & B_n \\ C_n & D_n \end{pmatrix}\begin{pmatrix} p_{II} \\ q_{II} \end{pmatrix} \quad (8)$$

where subscripts of the matrix elements corresponds to the individual resonator longitudinally arranged and its rear duct which are numbered as shown in Fig.1(a). The above p_I, q_I, p_{II}, and q_{II} are written as

$$p_I = p_{Ii} + p_{Ir} \quad (9)$$

$$q_I = \frac{1}{Z_0}\left(p_{Ii} - p_{Ir}\right) \quad (10)$$

$$p_{II} = p_{IIi} \quad (11)$$

$$q_{II} = \frac{p_{IIi}}{Z_0} \quad (12)$$

where p_{Ii} and p_{Ir} are, respectively, the incident and reflected pressures at the silencer entrance and p_{IIi} transmitted pressure in the tail duct.

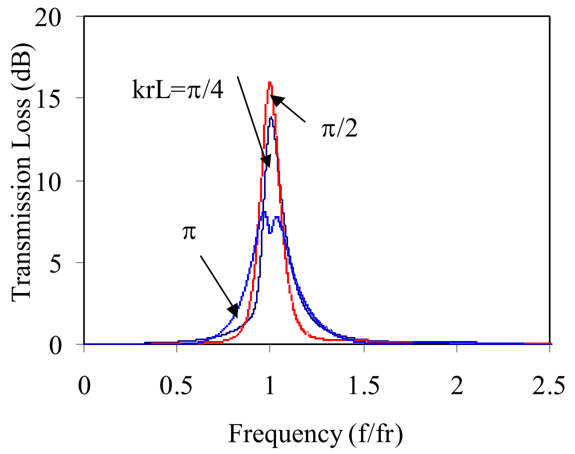

Fig.4 Effect of the resonance phase angle on the transmission loss characteristic.

The transmission loss TL is defined as

$$TL = 10\log\left|\frac{p_{Ii}}{p_{IIi}}\right|^2 \quad (13)$$

Each matrix for the resonator and partial duct may be obtained from eqs. (1)-(7), so that substituting eqs. (9)-(12) into eq. (8) and by removing p_{Ir}, one may get the acoustic energy ratio $|p_{Ii}/p_{IIi}|^2$. Then, using eq. (13), one could calculate the transmission loss for this resonance type silencer.

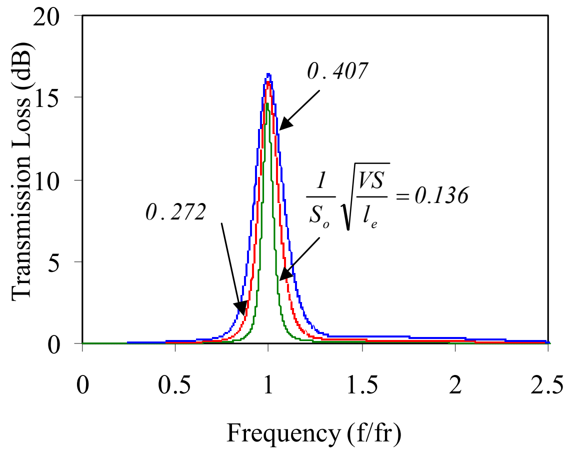

Fig.5 Effect of the geometric parameter on the transmission loss characteristic.

3. Typical results and conclusions

A comprehensive computer program has been prepared in FORTRAN for prediction of the transmission loss values, making use of the expressions derived above. Parametric studies were made for the default configuration as listed in Appendix A.

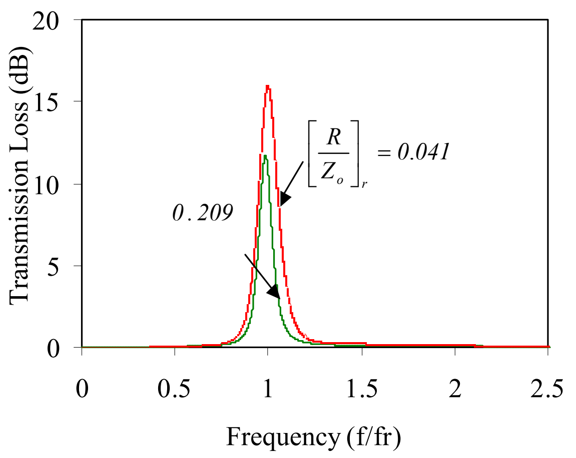

Fig.6 Effect of the resistance/duct impedance ratio on the transmission loss characteristic.

The resistance/duct impedance ratio at the resonance frequency $[R/Z_o]_r$ is related to the maximum attenuation of the silencer given by the physical quantities at room temperature in eq. (4). The geometric parameter $\sqrt{VS/l_e}/S_o$ influences the resonator reactance and this in turn widens the frequency range in which the silencer will efficiently function, however, it is disassociated from the attenuation at the resonance frequency, as it vanishes at $f=f_r$ as seen in eq.(5). The resonance phase angle k_rL, determined by substituting f_r in terms of frequency f in eq.(6), is much associated with the silencer performance. Namely, as shown in Fig.2, when it is $\pi/2$, the maximum transmission loss expressed by $[TL]_r$ becomes largest, decreasing as its value approaches zero or π. In the case of this investigation, the above parameters are fixed in each

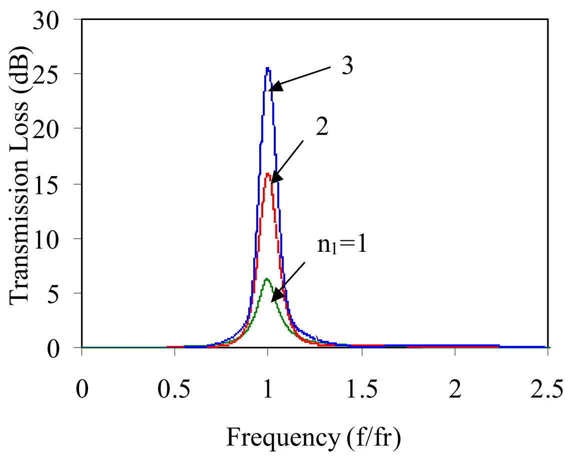

Fig.7 Effect of the quantity of resonators along duct on the transmission loss characteristic.

presented datum.

Figure 3 shows the numerical values of $[TL]_r$ against Mach number. They vary with smaller gradients as the flow speed increases and will almost be unchangeable with the Mach number exceeding about 0.15. Thus the multieffect of resonators is markedly significant at considerably lower Mach number and, according to the calculations, may be kept to a certain extent up to the limit of the incompressible flow even though it decreases.

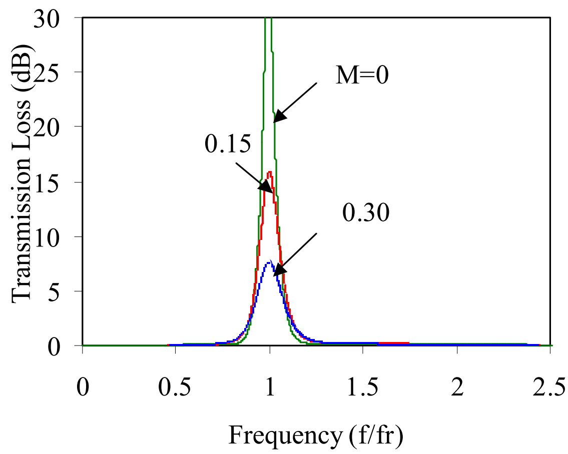

Fig.8 Effect of the Mach number on the transmission loss characteristic

These parametric studies indicate that transmission loss would improve with:

(i) resonance phase angle of $\pi/2$ as indicated by Fig. 4

(ii) more transverse resonators as is obvious from Fig. 5

(iii) lower resistance/duct impedance ratio as is clear from Fig. 6

(iv) more longitudinal resonators as is borne out by Fig. 7

(v) lower flow speed as is indicated by Fig. 8.

Appendix A

Duct dimension	51mmx51mm
Resonance chamber volume	$V=6.283\times10^{-6}$ m^3
Connector diameter	d=10 mm
Connector length	l=10 mm
Number of longitudinal resonators	n=2
Number of upper side transverse resonators	$m_1=2$
Number of lower side transverse resonators	$m_2=2$
Flow velocity	U=51.555 m/s
Resonance phase angle	$k_rL=\pi/2$
Ambient temperature	$t=20^{\circ}C$
Resonance frequency	$f_r=1540$ Hz

References

[1]. D. Davis, Jr., G. Stevens, Jr., D. Moore and G. Stokes,“Theoretical and Measured Attenuation of Mufflers at Room Temperature without Flow, with Comments on Engine-exhaust Muffler Design,” NACA Technical Note 2893, 1954.

[2]. Y. Hirata and T. Ito, “Influence of Gas Flow on Sound Attenuation due to a Resonator in a Duct,” J. Acoust. Soc. Japan **26**(1970) No.1 pp.16-24.

[3]. M. Hototzuka, “Characteristics for the Helmholtz Type Silencer,” Thesis for Master of Engineering, Tokai University 2000. (in Japanese)

[4]. P. Mungur and G.M.L. Gradwell, “Acoustic Wave Propagation in a Sheared Fluid Contained in a Duct,” J. Sound Vib., **9**(1969) No.1 pp.28-48.

[5]. L.E. Kinsler, A.R. Frey, A.B. Coppens, and J.V. Sanders, “Fundamentals of Acoustics,”John Wiley & Sons, 2000.

[6]. R.J. Alfredson and P.O.A.L. Davies, “Performance of Exhaust Silencer Components,” J. Sound Vib., **15**(1971) No.2 pp.175-196.

[7]. M.L. Munjal, “Velocity Ratio-cum-Transfer Matrix Method for the Evaluation of a Muffler with Mean Flow,” J. Sound Vib., **39**(1975) No.1 pp.105-119.