The 22nd Conference of Mechanical Engineering Network of Thailand
15-17 October 2008, Thammasat University, Rangsit Campus, Pathum Thani, Thailand

J-Integral Estimation for a Semi-Elliptical Surface Crack in Round Bar under Tension

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Abstract

The *J*-integral and ΔJ is an essential parameter for characterize material's fracture toughness and fatigue crack propagation rate in elastic-plastic regime, respectively. Round bar specimen provides a cost-effective way in utilizing a test material for evaluating the fatigue crack propagation rate both in elastic and elastic-plastic regimes. However, the available *J*-integral for a semi-elliptical surface crack in round bar under tension is still incomplete. This paper applies the reference stress method to estimate a *J*-integral and ΔJ for this problem. The estimation results are validated by numerical solutions taken from a literature. Based on a limited amount of the available solutions, the reference stress method can predict *J*-integral and ΔJ within an accuracy of 20 percents.

Key words : *J*-integral, reference stress method, semi-elliptical surface crack, round bar

1. Introduction

To assess the integrity of a cracked structure, it is important to know the solution of a crack tip parameter corresponding to that configuration. In linear elastic and elastic-plastic fracture analysis, the accepted parameters that characterize the crack tip condition are stress intensity factor *K* and *J*-integral, respectively. There is a large number of *K*-solution available for various configurations of structures, cracks and loads [1-3]. On the contrary, the number of *J*-integral solutions is much less available [4].

Round bar is one of a typical shape of structural element such as pins, bolts, reinforcement wires and shafts. In service, cracks may initiate from surface and grow across the section. To assess the fracture condition and correlate crack propagation rate, the relevance solution of crack tip parameter should be derived.

From the testing aspect, Shin et al. [5] has shown that a miniature surface cracked cylindrical rod specimen offer a cost-effective way for evaluating fatigue crack propagation rate in a small-scale yielding regime, as compare to commonly used specimen such as compact tension type.

Under large-scale plasticity condition, fatigue crack propagation rate can be successfully correlates with a parameter ΔJ, which can be derived from a *J*-integral solution. Findley [7] calculated the *J*-integral for this case using finite element analysis. However, the reported solution was limited to the case of crack depth to diameter equal to 0.246 and crack aspect ratio ranges from 0.5 to 1.5. In order to make a correlation between ΔJ and fatigue crack propagation rate, a wider range of *J*-integral solution is necessary.

There are several methods for deriving a *J*-integral solution. A nonlinear finite element analysis (FEM) has been received more attention at present [4,8]. This technique provides an accurate result if properly models the problem. However, it is a time consuming process especially in mesh generation [4]. One of the most attractive methods for estimating a *J*-integral is the reference stress method (RSM). This method was firstly proposed by Ainsworth [9]. Kim el al. [10-12] has been extensively applied and modified this method to several structure types, load and crack configurations. Validation of the solutions obtained by RSM with those obtained by FEM showed an acceptable accuracy in most cases. The RSM has been adopted in the structural integrity assessment codes, for examples R5 [13] and R6 [14].

In this paper, the RSM is used to estimate a *J*-integral solution of a semi-elliptical crack in round bar under tension. Review of RSM is present in section 2. Derivation of the *J*-integral and ΔJ for the studied case is presented in section 3. Comparisons are then made with finite element (FE) solutions taken from the literature [7] in section 4.

2. Reference stress method

This section details a derivation of a general form of the *J*-integral estimation equation by the RSM.

Consider a material that its deformation behavior can be expressed by the Ramberg-Osgood equation, i.e. Eq. (1).

$$\frac{\varepsilon}{\varepsilon_Y} = \frac{\sigma}{\sigma_Y} + \alpha\left(\frac{\sigma}{\sigma_Y}\right)^n \tag{1}$$

where ε_Y is yield strain, σ_Y is yield stress, α and n are strain hardening coefficient and exponent, respectively.

The general form of plastic component of J-integral for this type of material can be written as [4]

$$J_{pl} = \alpha\sigma_Y\varepsilon_Y g_1 h_1\left(\frac{P}{P_L}\right)^{n+1} \tag{2}$$

where g_1 is dimensionless function depends on geometry of structure, h_1 is dimensionless function depends on strain hardening exponent n and crack configuration, P is applied load and P_L is fully plastic load.

Rearrange Eq. (2) into the following form

$$J_{pl} = \alpha\sigma_Y\varepsilon_Y g_1 h_1\left(\frac{P_{ref}}{P_L}\right)^{n+1}\left(\frac{P}{P_{ref}}\right)^{n+1} \tag{3}$$

where P_{ref} is reference load

Equation (3) can be rewritten as

$$J_{pl} = \alpha\sigma_Y\varepsilon_Y g_1 h_1^*\left(\frac{P}{P_{ref}}\right)^{n+1} \tag{4}$$

where $h_1^* = h_1\left(\frac{P_{ref}}{P_L}\right)^{n+1}$ (5)

The proper value of P_{ref}, is that causes the values of h_1^* becomes independent of the strain hardening exponent.

Similary, from Eqs. (1) and (2), the general form of elastic component of J-integral is

$$J_{el} = \sigma_Y\varepsilon_Y g_1 h_1^*\left(\frac{P}{P_{ref}}\right)^2 \tag{6}$$

Divide Eq. (2) by Eq.(6) yields

$$\frac{J_{pl}}{J_{el}} = \alpha\left(\frac{P}{P_{ref}}\right)^{n-1} \tag{7}$$

Define the reference stress σ_{ref} as

$$\sigma_{ref} = \frac{P}{P_{ref}}\sigma_Y \tag{8}$$

Substitute into Eq.(7) yields

$$\frac{J_{pl}}{J_{el}} = \alpha\left(\frac{\sigma_{ref}}{\sigma_Y}\right)^{n-1} \tag{9}$$

From Eq.(1), the plastic component of the reference strain ε_{ref}^{pl} at $\sigma = \sigma_{ref}$ is

$$\varepsilon_{ref}^{pl} = \alpha\varepsilon_Y\left(\frac{\sigma_{ref}}{\sigma_Y}\right)^n \tag{10}$$

Substitute into Eq.(9) yields

$$\frac{J_{pl}}{J_{el}} = \frac{\sigma_Y}{\varepsilon_Y}\frac{\varepsilon_{ref}^{pl}}{\sigma_{ref}} \tag{11}$$

The term σ_Y/ε_Y is a modulus of elasticity E; thus

$$\frac{J_{pl}}{J_{el}} = \frac{E\varepsilon_{ref}^{pl}}{\sigma_{ref}} \tag{12}$$

The reference strain ε_{ref} is composed of elastic component ε_{ref}^{el} and plastic component ε_{ref}^{pl} ,i.e.

$$\varepsilon_{ref} = \varepsilon_{ref}^{el} + \varepsilon_{ref}^{pl} \tag{13}$$

Substitute into Eq.(12) yields

$$\frac{J_{pl}}{J_{el}} = \frac{E\varepsilon_{ref}}{\sigma_{ref}} - \frac{E\varepsilon_{ref}^{el}}{\sigma_{ref}} \tag{14}$$

But $\varepsilon_{ref}^{el}/\sigma_{ref} = 1/E$, therefore

$$1 + \frac{J_{pl}}{J_{el}} = \frac{E\varepsilon_{ref}}{\sigma_{ref}} \tag{15}$$

The J-integral is composed of elastic component J_{el} and plastic component J_{pl}, i.e. $J = J_{el} + J_{pl}$, thus

$$\frac{J}{J_{el}} = \frac{E\varepsilon_{ref}}{\sigma_{ref}} \tag{16}$$

From the relationship between J_{el} and K, Eq.(16) becomes

$$J = \frac{E\varepsilon_{ref}}{\sigma_{ref}}\left(\frac{K^2}{E'}\right) \tag{17}$$

$$\text{where } E' = \begin{cases} E & ; \textit{at surface} \\ E/\left(1-\nu^2\right) & ; \textit{otherwise} \end{cases} \tag{18}$$

To estimate a J-integral using Eq.(18), it requires to know the K-solution and P_{ref}. The P_{ref}, can be determined if how the value of h_1 varies with n is known. However, if this is not the case, the fully plastic load P_L could be used instead of P_{ref} with an acceptable accuracy for estimating a J-integral [15].

Equation (18) can be directly extended to a cyclic loading case to determine the cyclic J-integral or ΔJ as follows

$$\Delta J = \frac{E\Delta\varepsilon_{ref}}{\Delta\sigma_{ref}}\left(\frac{\Delta K^2}{E'}\right) \tag{19}$$

where $\Delta\varepsilon_{ref}$, $\Delta\sigma_{ref}$ are reference strain and reference stress ranges, respectively, and ΔK is stress intensity factor range.

3. *J*-integral estimation

The geometrical variables of a rod and crack are presented in figure 1. The crack front is modeled as elliptical arc having a center at (0,-D/2). The shaded area represents the crack area.

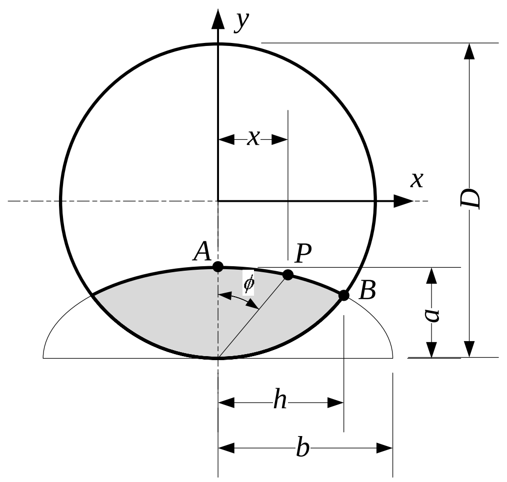

Fig. 1 Nomenclature used for an elliptical surface crack

The crack area A_c is determine from

$$A_c = \int_0^{x_B} \left[\sqrt{\left(1 - \frac{x^2}{b^2}\right) a^2} - \frac{D}{2} + \sqrt{\frac{D^2}{4} - x^2} \right] dx \quad (20)$$

where x_B is an abscissa of the intersection point between crack front and rod's perimeter.

Under a tension load, the fully plastic load P_L is

$$P_L = \sigma_Y \left(A - A_c\right) \quad (21)$$

where A is the cross-section area of the rod.
Substitute into Eq. (8) yields

$$\sigma_{ref} = \sigma \frac{A}{A - A_c} \quad (22)$$

For Ramberg-Osgood type material, the reference strain calculates from Eq.(1) is

$$\varepsilon_{ref} = \frac{\sigma_{ref}}{E} + \alpha \varepsilon_Y \left(\frac{\sigma_{ref}}{\sigma_Y} \right)^n \quad (23)$$

The stress intensity factor for this configuration [6] is

$$K = \sigma \sqrt{\pi a} F \quad (24a)$$

and $$F = \sum_{i=0}^{2} \sum_{j=0}^{7} \sum_{k=0}^{2} M_{ijk} \left(\frac{a}{b}\right)^i \left(\frac{a}{D}\right)^j \left(\frac{x}{h}\right)^k \quad (24b)$$

The coefficients M_{ijk} has been reported in Ref.[6] and is not repeated here.
Substituting Eqs. (22)-(24) into Eq.(17) yields

$$J = \left[1 + \alpha \left(\frac{\sigma}{\sigma_Y} \frac{A}{A - A_c} \right)^{n-1} \right] \left(\frac{\pi \sigma^2 a F^2}{E'} \right) \quad (25)$$

Under cyclic loading, the relationship between stress range $\Delta\sigma$ and strain range $\Delta\varepsilon$ could be expressed by the following form.

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\alpha \left(\frac{\Delta\sigma}{2\sigma_{Yc}} \right)^n \quad (26)$$

where σ_{Yc} is cyclic yield stress.

With the same procedure as described in this section, the following ΔJ estimation equation is obtained.

$$\Delta J = \left[1 + \frac{E\alpha}{\sigma_{Yc}} \left(\frac{\Delta\sigma}{2\sigma_{Yc}} \frac{A}{A - A_c} \right)^{n-1} \right] \left(\frac{\pi \Delta\sigma^2 a F^2}{E'} \right) \quad (27)$$

4. Validation and Discussion

In this section, the J-integral and ΔJ are calculated using Eqs. (25) and (27), respectively. The results are then compared with the available FE solutions.

The material parameters used in computation were $\alpha = 0.00049$, $n = 8$, $\sigma_Y = 1630$ MPa, $\sigma_{Yc} = 815$ MPa, and $E = 210.8$ MPa. However, a Poisson's ratio ν for this material is not available; therefore, $\nu = 0.3$ is assumed.

The rod diameter is 6.35 mm and the ratio of crack depth to rod diameter is 0.246.

The J-integral at a deepest point for several of crack aspect ratios under an applied stress of 1240 MPa is shown in Fig. 2. This figure shows that J-integral obtained by RSM accurate within 20 percent of that obtained by FEM.

Variation of J-integral along the crack front is computed by varying the distance x. Figure 3 shows the results from a point of maximum depth to a point near surface for a case of $a/b = 0.5$.

Figures 4 and 5 compare the ΔJ estimated by RSM to that obtained by FEM, at a point of maximum depth and at a point of 45° from a maximum depth, respectively. The applied stress range is 1510 MPa. These figures show that the RSM can estimate ΔJ with an accuracy of 20 percent.

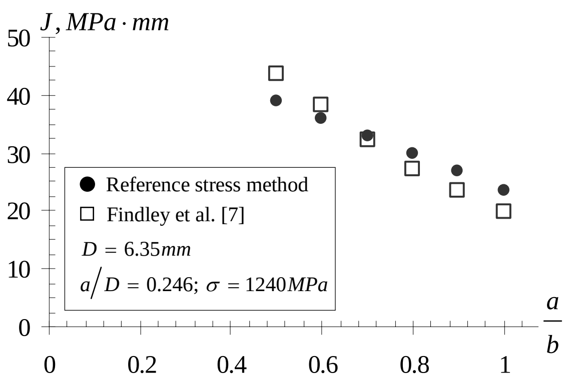

Fig. 2 Dependence of J-integral on crack aspect ratio at the maximum depth.

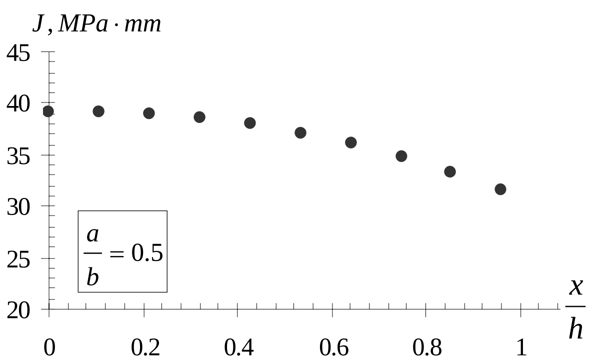

Fig. 3 Dependence of J-integral on position along the crack front estimated by RSM.

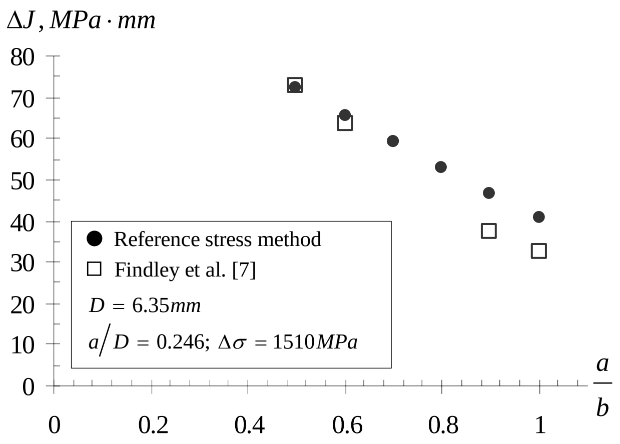

Fig. 4 Dependence of ΔJ on crack aspect ratio at the maximum depth.

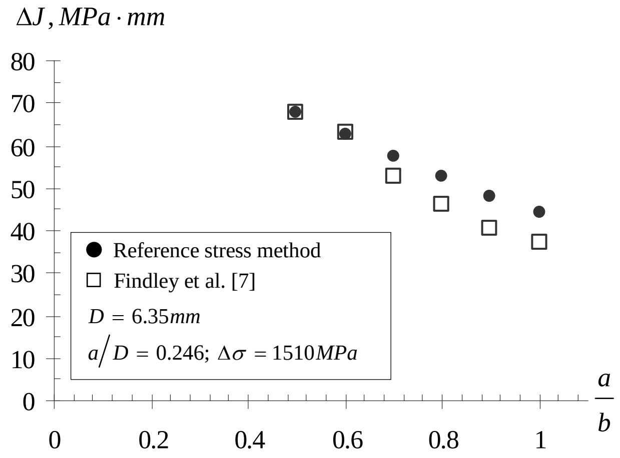

Fig. 5 Dependence of ΔJ on crack aspect ratio at 45 deg. from the maximum depth.

Even the reference stress based on global yielding load can estimate an acceptable accurate *J*-integral and ΔJ, the extensive finite element solutions for a various strain hardening exponent *n* is still necessary. Because, a more suitable reference load is required for improving accuracy of *J*-integral estimation, especially in the case of material that is not obey the Ramberg-Osgood behavior.

5. Conclusion

In this paper, derivation of the equations for estimating *J*-integral and ΔJ by reference stress method are presented. These equations are then applied to a round bar having semi-elliptical surface crack. Comparisons have been made between solutions obtained by RSM with those obtained by FEM. Within the range of studied, it has been found that the RSM can estimate the *J*-integral and ΔJ with an accuracy of 20 percent or better.

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