Design for Robust Stability and Performance Using Fuzzy Gain Scheduling Based on Robust Stability Theorem

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Abstract

Using class gamma robust stability analysis theorem, we present in this paper design of fuzzy gain scheduling controller for linear systems subjected to time-varying nonlinear uncertainties in system parameters. Unlike most gain scheduling techniques, ours is special in the sense that it addresses both robust stability and robust performance at the same time. It is also free from restrictions on initial conditions, time rate of uncertainties, and time rate of gain scheduling. Here, we provide design procedure for the general case, and apply it in an example to obtain a PID controller with differential gain scheduling for independent joint control of a modified SCARA robot arm. Numerical simulations showed that the resulting tracking errors and oscillations were satisfactorily small when subjected to typically large uncertainties.

Keywords

Stability, Class Gamma, PID, Gain Scheduling, Robot

I. Introduction

A common characteristic of many gain scheduling controllers is that they offer stability and performance for linearized approximating models of a nonlinear dynamical system about equilibrium points or family of equilibrium points. However, stability of linearized models about such points does not guarantee stability of the corresponding nonlinear system in general [1]. Accordingly, gain scheduling is a controller design technique that focuses more on practical applications than theoretical rigors. Despite this common serious weakness, gain scheduling is the technique of choice in many demanding and extremely complex uncertain nonlinear systems such as fighter aircrafts [2], and fuel injected internal combustion engines [3]. This is generally because a more desirable nonlinear solution could not be found.

Fuzzy logic has been employed for decades in many realistic applications to transform proven linguistic knowledge and reasonable opinions to the corresponding numerical outputs. In particular, many gain scheduling schemes for PID controllers were based on fuzzy logic, see [4, 5] and references therein. Despite of its popularity in the control community, the mapping it provides is usually extremely nonlinear. Accordingly, stability of the resulting control system is usually very difficult to show analytically. Although many investigations on this matter were conducted using numerical simulations [4-6], these were insufficient for predicting complex behaviors of nonlinear systems in general.

Robust stability analysis (RSA) theorems were employed to address stability of time-varying uncertain linear systems in [7-9]. RSA theorems belonging to class gamma [10] found applications in controller design with exphasis of robust stability in [10-11]. This RSA theorem was extended to address simultaneously robust stability and robust performance for the same class of systems by means of gain scheduling in a general setup [12]. While necessary theoretical foundations for robust performance were given there, the paper did not propose a specific approach for it. This paper is then written to propose specifically the use of fuzzy logic to achieve robust stability and performance in details. The resulting fuzzy gain scheduling PID controller is guaranteed to be stable and is free from restrictions on initial conditions and time rate of scheduled variables usually found in many gain scheduling techniques.

2. Mathematical Description

In this paper, we are interested in computing gain scheduling control laws that guarantee input-to-state stability for linear systems with time-varying nonlinear uncertainties in system parameters:

$$\dot{x} = [A + \Delta A(x,t)]x - [B + \Delta B(x,t)][K_n + \Delta K(x,t)]x + f(x,u) \quad (1)$$

where $x \in \Re^n$ is the state vector, the system matrix $A \in \Re^{n\times n}$ is known, the input matrix $B \in \Re^{n\times m}$ is known, $K_n \in \Re^{m\times n}$ is the nominal state feedback gain matrix to compute, the state-independent input $u \in \Re^p$ is unknown, the bounded nonlinear uncertain perturbation $f(x,u) \in \Re^n$ is unknown, and Δ denotes time-varying nonlinear uncertainties with appropriate dimensions and known bounds on matrices *A* and *B*. Special attention should be drawn towards $\Delta K(x,t)$ which represents uncertainties associated with the nominal state feedback gain matrix K_n of the system of interest. Note that the origin is the equilibrium point of Eq.(1), whose right hand side is continuously differentiable and is globally Lipschitz in *x* and *u*, uniformly in *t*. A nonzero equilibrium point can always be shifted to the origin by redefining relevant state variables appropriately.

3. Fuzzy Gain Scheduler

Fuzzy logic theory allows plausible mathematical representations of multi-valued logics that humans use for decision-making and reasoning. By carefully observing characteristics of system responses when subjected to various inputs and disturbances over reasonable periods of time, it may be possible to select linguistic variables that capture or dominate system dynamics. Then for a linguistic variable, we define membership functions to describe it in fuzzy domains. From that we define fuzzy rules and thus fuzzy rule base for gain scheduling, which represent one's knowledge on dynamics of the system of interest.

We propose that the uncertain matrix $\Delta K \in \Re^{m\times n}$ is composed of two components:

$$\Delta K = \Delta K_u + \Delta K_p \quad (2)$$

where ΔK_u is the component of "true" uncertainties associated with the nominal gain matrix K_n and the corresponding available specifications, and ΔK_p is the component of "psudo"-uncertainties that is scheduled by using fuzzy logic. We manage robust stability and performance of the resulting control system through the nominal gain matrix K_n and the scheduled gain matrix ΔK_p respectively.

We adopt for our presentation definitions and results relating to fuzzy logic in [13]. Here, the q^{th} fuzzy IF-THEN rule for scheduling $\Delta K_p(k,l)$ - the (k,l) element of ΔK_p, is given by:

$$\text{IF } \{ v_1 \text{ is } F_1 \text{ and ... and } v_p \text{ is } F_p \}_q$$
$$\text{THEN } \{ \Delta K_p(k,l) \text{ is } T(k,l) \}_q$$

where v_g, g = 1, ..., p is the g^{th} linguitic scheduling variable, *p* is the number of linguistic variables, F_g is a fuzzy set associated with v_g, and $T(k,l)$ is a fuzzy set associated with $\Delta K_p(k,l)$. The membership value of the IF part is computed using the product rule:

$$\mu_{IF} = \mu_1(v_1)..\mu_p(v_p)$$

where μ_g, g = 1, ..., p is the g^{th} membership function of fuzzy set in the IF part that describes v_g for which we impose that $0 \le \mu_g \le 1$. Now let f_g be the center of the fuzzy set $T(k,l)$ associated with the THEN part of the q^{th} fuzzy rule, we employ center-average defuzzifier to compute $\Delta K_p(k,l)$ according to the equation:

$$\Delta K_p(k,l) = \left(c\frac{\sum_{q=1}^{w} f_q\left(\mu_1(v_1)...\mu_p(v_p)\right)_q}{\sum_{q=1}^{w}\left(\mu_1(v_1)...\mu_p(v_p)\right)_q} \right)_{k,l} \quad (3)$$

where $c \in \Re$ is the scaling factor for $\Delta K(k,l)$, and *w* is the number of fuzzy rules in the fuzzy rule base for scheduling $\Delta K(k,l)$. We allow adaptation of fuzzy parameters at any rate, provided that the adaptation law is continuously diffirentiable and are globally Lipschitz as stated in the previous section.

4. A Class Gamma Theorem

When ignoring the unknown perturbation vector $f(x,u) \in \Re^n$ and all structed specifications associated with uncertain matrices ΔA, ΔB, and ΔK are available, it is always possible to write the interested equation of motion in the following form:

$$\dot{x} = \bar{A}x + \sum_{j=1}^{r}\left[h_j(x,t)E_j\right]x \quad (4)$$

where $\bar{A} \equiv A - BK$ is known, $E_j \in \Re^{n\times n}$ is known, and $h_j(x,t) \in \left[h_{lj}, h_{uj}\right]$ is a time-varying nonlinear uncertain scalar function with known bounds. We require that the uncertainty specifications E_j, h_{uj}, and h_{lj} are known for all uncertain elements in the control system. For convenience, we now provide the reader a class-gamma robust stability theorem from [12] that states sufficient conditions for exponential stability of the system of interest in Eq.(4). A proof for this theorem can be found in [12].

Theorem 1 [12] If the dynamical system in Eq.(4) is continuously differentiable and is globally Lipschitz with matrix $\bar{A}$ being Hurwitz and

$$max(\lambda(Z)) < 0 \quad (5)$$

where $max(\lambda(Z))$ is the maximum eigenvalue of *Z*, then the equilibrium point at the origin is globally exponentially stable. The matrix $Z = Z^T \in \Re^{n\times n}$ is obtained by:

1) Specified Q > 0 and $\bar{A}$ to compute P from the Lyapunov equation $-Q = (1/2)\left[P\bar{A} + \bar{A}^T P\right]$.
2) Compute $\bar{A}_l = \bar{A} + \sum_{j=1}^{r} h_{lj}E_j$, and $\Phi = P\bar{A}_l + \bar{A}_l^T P$.
3) Compute $\Psi_j = \left[PE_j + E_j^T P\right] = \Psi_j^T$.
4) Compute $\Lambda_{\Psi_j} = \mathrm{T}_{\Psi_j}^T \Psi_j \mathrm{T}_{\Psi_j} = \mathrm{diag}[\lambda_{\Psi_j 1} \ \ldots \ \lambda_{\Psi_j n}]$, where

$T_{\Psi_j} = \left[v_{\Psi_j 1} \mid \cdots \mid v_{\Psi_j n} \right]$, and $\left\{ v_{\Psi_j 1}, \ \ldots \ , v_{\Psi_j n} \right\}$ is the set of n orthonormal eigenvectors of Ψ_j.

5) Compute $\Lambda_{\Psi_j}^{\geq 0}$ by setting all negative elements of Λ_{Ψ_j} to zero

6) Compute $\Psi_j^{\geq 0} = T_{\Psi_j} \Lambda_{\Psi_j}^{\geq 0} T_{\Psi_j}^T$.

7) Compute $Z \equiv \Phi + \sum_{j=1}^{r} \left[\left(h_{uj} - h_{lj} \right) \Psi_j^{\geq 0} \right]$.

By Lyapunov stability theorem, exponential stability of the unperturbed system implies that the perturbed system is input-to-state stable, that is bounded inputs produce bounded states [1]. We employ this theorem to determine allowable bounds on true uncertainties and psudo-uncertainties. The latter leads to a sufficient condition for input-to-state stability of our fuzzy gain scheduling control system discussed in the next section.

5. Stability of Fuzzy Gain Scheduling System

Stability of the fuzzy gain scheduling system is addressed as Lemma 1 in the following:

Lemma 1 For the fuzzy gain scheduling system proposed previously, suppose Theorem 1 is satisfied with a set of upper and lower bounds on all elements of $\Delta A, \Delta B, \Delta K_u$ and ΔK_p then system is input-to-state stable if for all k = 1, ..., m, and l = 1, ..., n, the fuzzy rule base for scheduling $\Delta K_p(k,l)$ is such that:

$$\left. \begin{aligned} h_u(k,l) &\geq c(k,l) \max_{q=1}^{w} \left(f_q(k,l) \right) \\ h_l(k,l) &\leq c(k,l) \min_{q=1}^{w} \left(f_q(k,l) \right) \end{aligned} \right\} \tag{6}$$

where $h_u(k,l)$ and $h_l(k,l)$ are the allowable upper and lower bounds on $\Delta K_p(k,l)$ respectively, and w is the number of fuzzy sets.

Proof By Theorem 1, if all the upper and lower bounds on all elements of $\Delta A, \Delta B, \Delta K_u$ and ΔK_p are such that the theorem is satisfied, then the fuzzy gain scheduling system is guaranteed to be input-to-state stable. It then remains to show for all k = 1, ..., m, and l = 1, ..., n that the fuzzy rule base for $\Delta K_p(k,l)$ is such that:

$$h_l(k,l) \leq \left(c \frac{\sum_{q=1}^{w} f_q\left(\mu_1(v_1) \ldots \mu_p(v_p) \right)}{\sum_{q=1}^{w} \left(\mu_1(v_1) \ldots \mu_p(v_p) \right)} \right)_{k,l} \leq h_u(k,l)$$

Now, notice for the center-average defuzzifier that

$$\left. \min_{q=1}^{w} \left(f_q \right) \right|_{k,l} \leq \left(\frac{\sum_{q=1}^{w} f_q\left(\mu_1(v_1) \ldots \mu_p(v_p) \right)}{\sum_{q=1}^{w} \left(\mu_1(v_1) \ldots \mu_p(v_p) \right)} \right)_{k,l} \leq \left. \max_{q=1}^{w} \left(f_q \right) \right|_{k,l}$$

Accordingly, the fuzzy rule base is as required if the two conditions in the theorem are satisfied simultaneously. This completes the proof.

Basically, Lemma 1 states that if Theorem 1 is satisfied with a set of upper and lower bounds on all elements of $\Delta A, \Delta B, \Delta K_u$, and ΔK_p then the system is input-to-state stable if the centers of all fuzzy sets in the THEN part of all fuzzy IF-THEN rules are bounded by those associated with appropriate element of ΔK_p. The shape of membership functions do not affect stability of the fuzzy gain scheduling system.

6. Design of Fuzzy Gain Scheduling System

We propose that design of our fuzzy gain scheduling system is separated into two parts. This former is for robust stability and the latter is for robust performance.

Part I: Robust Stability

We propose the following two-step design for robust stability

1. Apply existing linear robust control techniques to find the nominal gain matrix K_n such that the necessary condition of $\left[A - BK_n \right]$ being Hurwitz is satisfied.
2. Employ Theorem 1 to find all allowable bounds on uncertain elements of $\Delta A, \Delta B, \Delta K_u$ and ΔK_p.

Note for this part that allowable bounds resulting from an existing robust control technique may be more conservative than those resulting from others. We generally require that these bounds are the least conservative because they imply strong robustness for the system, and allows large variation for ΔK_p. Typically, we find that the procedure in [11] can be employed to accomplish the above two steps simultaneously such that the resulting allowable bounds are larger than required.

Part II: Robust Performance

The system is guaranted to be input-to-state stable under any scheduling scheme on $\Delta K_p(k,l)$, provided that it obeys the corresponding upper and lower bounds determined in Part I. Here, we do gain scheduling using fuzzy logic primarily because it provides means to improve performance of the control system by using relevant human knowledge and reasoning in the form of fuzzy IF-THEN rules. Although a fuzzy scheduling scheme is usually specific to its applications. it can usually be captured by the following two IF-THEN rules:

R1. IF {the error is large and the error is increasing} THEN {the control action in the direction that forces the error to decrease should be strengthened}.

R2. IF {the error is large and the error is decreasing} THEN {the control action in the direction that forces the error to decrease should be weakened}.

where the degrees of "large", "increasing", "decreasing", "strengthened", and "weakened" are described by the relevant membership functions. Variation in degrees of these qualities leads to additional fuzzy rules and thus fuzzy rule bases.

7. Example

Consider the problem in which a DC actuating motor for joint 2 of a modified SCARA robot is controlled independently to track a time-varying trajectory [12]. Dynamics of the system can be represented by:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & -(3.8+h_{A1})\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0\\ 0\\ -(89.3+h_{B1})\end{bmatrix}V+\begin{bmatrix}0\\ 0\\ (\ddot{r}+4.25\dot{r}+254.6T_d)\end{bmatrix}$$

where state variable x_2 is trajectory error, $x_1=\int x_2 dt$, and $x_3=\dot{x}_2$, r is trajectory reference signal, T_d is disturbance torque resulting from coupled dynamics of manipulator links, h_{A1} and h_{B1} are uncertain constants that depend on manipulator mechanical properties. The manipulator is shown in Fig. 1.

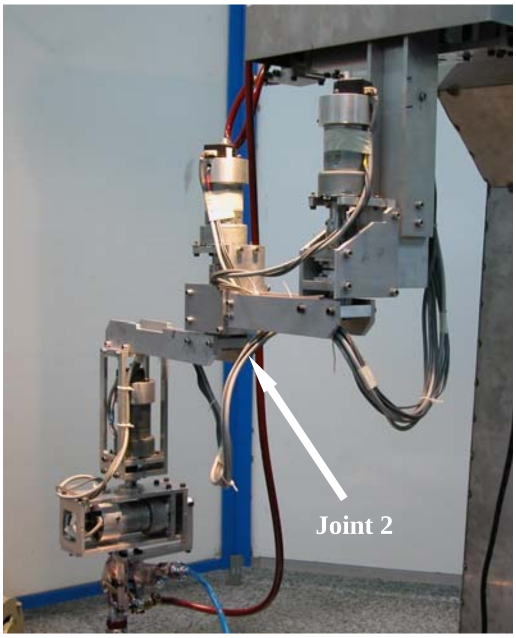

Fig. 1 The Modified SCARA Robot in the Example

The available uncertainty specifications associated with h_{A1} and h_{B1} are upper and lower bounds:

$$h_{A1}\in[0,\ 0.86]\equiv[h_{l,A1},\ h_{u,A1}]$$

$$h_{B1}\in[0,\ 20.52]\equiv[h_{l,B1},\ h_{u,B1}]$$

Now, let us employ the PID gain scheduling control law:

$$V=-K(x)x$$

where $K(x)=\left[K_i \mid K_p \mid K_d+\Delta K_d(x)\right]$, K_i, K_p and K_d are scalar constants, and $\Delta K_d(x)$ is the scheduled change of differential gain K_d. Using this control law, dynamics of the system is given by:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & -\begin{pmatrix}3.8+h_{A1}\\ +h_{A2}(x)\end{pmatrix}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0\\ 0\\ -(89.3+h_{B1})\end{bmatrix}K_n x$$

where the nominal gain matrix is $K_n\equiv\left[K_i \quad K_p \quad K_d\right]$, and

$$h_{A2}(x)\equiv-(89.3+h_{B1})\Delta K_d(x)$$

Note that we have accounted $\Delta K_d(x)$, the scheduled variation of the differential gain, as an additional uncertain element $h_{A2}(x)$ of the system matrix.

To achieve robust stability for the uncertain control system, we employ the given procedure to obtain the nominal state feedback gain matrix K_n that accounts for all uncertainties, and allowable bounds on $\Delta K_d(x)$:

$$K_n=[-6 \quad -10.4 \quad -5.98]$$

$$-5.0\le\Delta K_d(x)\le 0$$

with symmetric matrices relating to Theorem 1:

$$P=\begin{bmatrix}3.47 & 2 & 3.7\times10^{-3}\\ ... & 3.48 & 6.5\times10^{-3}\\ ... & ... & 3.7\times10^{-3}\end{bmatrix},\ Q=\begin{bmatrix}2 & 1.73 & 1\\ ... & 4.01 & 1.73\\ ... & ... & 1.99\end{bmatrix}$$

and $Z=\begin{bmatrix}-3.54 & -2.67 & -2.56\\ ... & -6.63 & -4.44\\ ... & ... & -3.28\end{bmatrix}$. The maximum eigenvalue of Z is -0.045. By Theorem 1, the uncertain system is input-to-state stable provided that $\Delta K_d(x)$ obeys the above bounds.

We now employ the preferred type of fuzzy gain scheduler:

$$\Delta K_d=c\frac{\sum_{q=1}^{25} f_q\left(\mu_1(x_1)\mu_2(x_2)\mu_3(x_3)\right)_q}{\sum_{q=1}^{25}\left(\mu_1(x_1)\mu_2(x_2)\mu_3(x_3)\right)_q}$$

where we set c = 1, and w = 25. Note that $\mu_g(x_g)$, g = 1, 2, 3 is the membership function of fuzzy set in the IF part that describes x_g. We set $\mu_1(x_1)=1$, while the membership functions and the corresponding fuzzy sets that describe x_2 and x_3 in the IF part are shown in Table 1. Notice that these membership functions are continuously differentiable and are globally Lipschitz in their variables. For this particular example, the membership function of a fuzzy set that describes x_2 is the same as the corresponding one for x_3, although this is not the case in general. We define these membership functions according to important characteristics of nominal system responses when subjected to various inputs and disturbances.

We put 25 fuzzy IF-THEN rules in the fuzzy rule base shown in Fig. 2. Note that the numbers in the rule base are the centers of membership functions of fuzzy sets for ΔK_d in the THEN part and not the fuzzy sets themself. Notice that all the centers are bounded by the lower and upper bounds on ΔK_d computed previously. Accordingly, the two conditions in Lemma 1 are satisfied simultaneously.

Table 1 Membership Functions of Relevant Fuzzy Sets

Fuzzy Sets	Membership Functions for x_g, g = 2, 3
NB	$\mu_g = 1/\left(1+e^{1000x_g+7}\right)$
NS	$\mu_g = e^{-81632.65(x_g+0.004)^2}$
Z	$\mu_g = e^{-2.5\times10^5 x_g^2}$
PS	$\mu_g = e^{-81632.65(x_g-0.004)^2}$
PB	$\mu_g = 1/\left(1+e^{-1000x_g+7}\right)$

Using the specified fuzzy differential gain scheduling, we run numerical simulations for the case in which all the initial conditions are zeroes, and

$$r = 78\left(t + 0.2\,sin(4t)\right),$$

$$T_d = 0.2\left(sin(10t)+cos(10t+2)\right),$$

Although h_{A1} and h_{B1} are uncertain constants, we impose the following nonlinear functions for them to pose additional difficulties for the controller:

$$h_{A1}(x) = \frac{0.86}{2}\left(sin(x_1)+1\right),$$

$$h_{B1}(x) = \frac{20.52}{2}\left(cos(x_3)+1\right).$$

Simulation results in Fig. 2 show $\Delta K_d(t)$ and $x_2(t)$ during $0 \le t \le 10s$ corresponding to the cases in which the fuzzy gain scheduler presents and absents. Notice for both cases that trajectory errors $x_2(t)$ are bounded. In addition, trajectory error corresponding to the former case is significantly less than that corresponding to latter during transient period of $0 \le t \le 2s$, and during $4 \le t \le 10s$. Responding speeds in both cases are similar.

		x_3				
		NB	NS	Z	PS	PB
x_2	NB	-4.784	-3.588	-2.99	0	0
	NS	-3.588	-3.588	-2.392	0	0
	Z	-2.99	-2.392	-2.392	-2.392	-2.99
	PS	0	0	-2.392	-3.588	-3.588
	PB	0	0	-2.99	-3.588	-4.784

Fig. 2 Fuzzy Rule for Scheduling ΔK_d

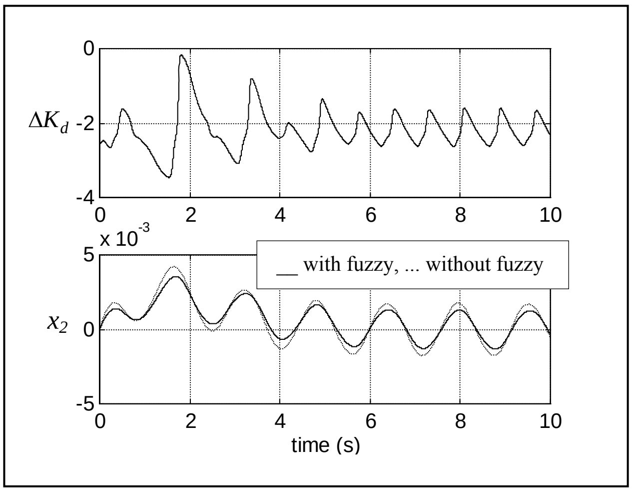

Fig. 3 Simulation Results of the fuzzy Gain Scheduling System

8. Conclusion

In this paper, we address the problem of robust stability and performance of linear systems subjected to time-varying nonlinear structured uncertainties in system parameters using fuzzy gain scheduling control. Under the assumption that all the relevant uncertainty specifications are available, we propose a two-steps robust controller design technique. In the first step, we put with existing uncertainties all variations of the scheduled gains as psudo-uncertainties, and then employ an appropriate class-gamma robust stability analysis theorem to determine a nominal linear control law that guarantees input-to-state stability for the uncertain system. In the second step, human knowledge and opinion on system dynamics is employed to increase performance through fuzzy gain scheduler, which is designed according to the relevant allowable bounds for the psudo-uncertainties obtained in the first step. Extensive numerical simulations confirm that the proposed fuzzy gain scheduling system is robustly stable when presented with the parameter uncertainties. When the control system is subjected to time-varying reference signal and external disturbances, the fuzzy gain scheduler can decrease tracking errors, and improve transient response characteristics significantly.

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