

THEORETICAL CHARACTERISTICS OF JOURNAL BEARINGS LUBRICATED WITH NON-NEWTONIAN LUBRICANTS INCLUDED THERMAL EFFECTS

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ABSTRACT

The thermohydrodynamic lubrication has been investigated to obtain the static and dynamic characteristics of journal bearings lubricated with non-Newtonian lubricants based on the Carreau viscosity model. The time-dependent modified Reynolds equation and the non-adiabatic energy equations have been formulated using the Carreau viscosity model. The simultaneous system of modified Reynolds and non-adiabatic energy equations included the heat conduction in the bearing bush were solved numerically with boundary conditions using finite difference technique. Static characteristics are presented for pressure distribution, temperature distribution, load carrying capacity and friction force with varying eccentricity ratio. The performance characteristics of journal bearings lubricated with non-Newtonian Carreau lubricants are compared with the characteristics of journal bearing with non-Newtonian lubricant using the Power-law model

Key words Non-adiabatic energy equation, Carreau viscosity model, Power-law model, Hydrodynamic journal bearings, Static characteristics of journal bearings.

1. Introduction

The performance characteristics of journal bearing with non-Newtonian behavior have been studied by a number of investigators. Two typical models; Power-law and Rabinowitsh model; are generally analyzed and proposed in many literatures. Hashimoto H.[3] and Williams P.D.[10] analyzed the lubrication equation by the Rabinowitsh model. Dien I.K.[4] derived the generalized steady-state Reynolds equation for the power-law rheology model and his work was extended by Jiin-Yuh Jang [5] to study the adiabatic solution for a finite width hydrodynamic journal bearing. Numerical solutions of Power-law lubricant were

presented by Buckolz R.H. [2] and by Wang J.[9]. Knight [6] and Jang [5] analyzed the non-adiabatic solution of journal bearings. Recently, Bouyahia F.[1] presented the influence of non-Newtonian character based on the Hershel-Bulkley, Bingham, Power-law and Rabinowisch models, and Yurusoy [8] used the Powell-Eyring model to study the pressure distribution in a slider bearing.

Based on the Power-law or Rabinowitsh model for the journal bearing for a given eccentricity ratio, shear-thinning effects tend to decrease the pressure, load capacity, friction force, and increase attitude angle. However, the relationships between shear rate and shear stress of the pseudoplastic fluids frequently appears to be a Newtonian fluid with very high viscosity at low shear rates and then to be a Newtonian fluid with lower viscosity at higher shear rates, and so the power-law or cubic equation model can't predict this non-linear behaviors accurately. The aim of this paper is to investigate the static and dynamic characteristics of journal bearing when the lubricants are in transition state from non-Newtonian to Newtonian behaviors by increasing the journal speed. In the study, the flow is assumed to be under laminar condition, and neglecting the inertia force. Carreau viscosity model is proposed to formulate the Reynolds and non-adiabatic energy equations for a finite width hydrodynamic journal bearing by using perturbation technique. Both equations were simultaneously solved using finite difference method.

2. Reynolds equation

The relationship between shear stress and shear rate for the non-Newtonian fluid can be represented by Carreau viscosity model as

$$\tau_{zx} = \mu \frac{\partial u}{\partial z} = \left\{ \mu_{\infty} + (\mu_0 - \mu_{\infty}) \left(1 + a^2 I \right)^{\frac{n-1}{2}} \right\} \frac{\partial u}{\partial z} \quad (1)$$

$$\tau_{zy} = \mu \frac{\partial v}{\partial z} = \left\{ \mu_{\infty} + (\mu_0 - \mu_{\infty}) \left(1 + a^2 I \right)^{\frac{n-1}{2}} \right\} \frac{\partial v}{\partial z} \quad (2)$$

where μ is apparent viscosity, μ_0 and μ_{∞} are limiting viscosity at very low and very high shear rates respectively. n is the power-law exponent which describes the slope of viscosity as a function of shear rate in the shear thinning regime and a is time constant. The apparent viscosity μ depends on the second invariant of the strain rate tensor I .

$$I = \left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \quad (3)$$

Adopting the perturbation method as shown in appendix, the velocity component u and v can be expanded as follows

$$u = \frac{U_J}{h} z + \frac{z(z-h)}{2} \frac{1}{G} \frac{\partial p}{\partial x} \quad (4)$$

$$v = \frac{z(z-h)}{2} \left\{ \frac{1}{\mu^*} \frac{\partial p}{\partial y} \right\} \quad (5)$$

where

$$I^* = \frac{(U_J - U_B)^2 - (V_J - V_B)^2}{h^2} \quad (6)$$

$$\mu^* = \mu_{\infty} + (\mu_0 - \mu_{\infty}) \left(1 + a^2 I^* \right)^{\frac{n-1}{2}} \quad (7)$$

$$G = \mu^* + 2 \left(\frac{\partial u^*}{\partial z} \right)^2 \left(\frac{\partial \mu}{\partial I} \right)_{I=I^*} \quad (8)$$

Using the continuity equation, we obtain the Reynolds equation for Carreau viscosity model.

$$\frac{\partial}{r \partial \theta} \left(\frac{h^3}{12G} \frac{\partial p}{r \partial \theta} \right) + \frac{\partial}{\partial y} \left(\frac{h^3}{12\mu^*} \frac{\partial p}{\partial y} \right) = \frac{U_J}{2} \frac{\partial h}{r \partial \theta} + \frac{\partial h}{\partial t} \quad (9)$$

Where the film thickness h is

$$h = c (1 + \varepsilon \cos \theta) \quad (10)$$

3. Non-Adiabatic energy equation

Under the non-adiabatic assumption and neglecting the temperature variation across the film thickness, the two dimensional energy equation for an incompressible fluid with laminar flow can be written as

$$\rho c_v \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \tau_{zx} \frac{\partial u}{\partial z} + \tau_{zy} \frac{\partial v}{\partial z} + \frac{\partial}{\partial z} \left(k_f \frac{\partial T}{\partial z} \right) \quad (11)$$

The film thickness is much smaller than other sizes of bearings, the integration of equation (11) can be approximated

using the mean velocities $u_m = \frac{1}{h} \int_0^h u dz$ and the mean

temperature $T_m = \frac{1}{h} \int_0^h T dz$, and written as

$$\begin{aligned} & \rho c_v h \left(\frac{\partial T_m}{\partial t} + u_m \frac{\partial T_m}{\partial x} + v_m \frac{\partial T_m}{\partial y} \right) \\ &= U_J \tau_{zx} \Big|_{z=h} - \frac{\partial p}{\partial x} \int_0^h u dz - \frac{\partial p}{\partial y} \int_0^h v dz + \left(k_f \frac{\partial T}{\partial z} \right)_{z=h} \\ & - \left(k_f \frac{\partial T}{\partial z} \right)_{z=0} \end{aligned} \quad (12)$$

where

$$\tau_{zx} = \left(\mu^* \frac{U_J}{h} \right) + \left(\frac{2z-h}{2} \frac{\partial p}{\partial x} \right) \quad (13)$$

The temperature gradients at the walls are obtained from the temperature profile across the film, which is assumed to be of second order

$$T = A + B \left(\frac{z}{h} \right) + C \left(\frac{z}{h} \right)^2 \quad (14)$$

Equation (14) may be written in terms of the journal surface temperature T_J , the local film temperature T_m , and the local inner bearing surface temperature T_{BI}

$$T = T_{Bl} + 2(3T_m - T_J - 2T_{Bl})\left(\frac{z}{h}\right) + 3(T_J - 2T_m + T_{Bl})\left(\frac{z}{h}\right)^2 \quad (15)$$

Then, the heat transfer by conduction is given by

$$\left(k_f \frac{\partial T}{\partial z}\right)_{z=h} - \left(k_f \frac{\partial T}{\partial z}\right)_{z=0} = \frac{6k_f}{h}(T_J - 2T_m + T_{Bl}) \quad (16)$$

For steady state condition, equation (12) can be written as

$$\rho c_v \left[\left(\frac{U_J h}{2} - \frac{h^3}{12G} \frac{\partial P}{r \partial \theta} \right) \frac{\partial T_m}{r \partial \theta} - \left(\frac{h^3}{12\mu^*} \frac{\partial p}{\partial y} \right) \frac{\partial T_m}{\partial y} \right] = \frac{6k_f}{h}(T_J - 2T_m + T_{Bl}) + \mu^* \frac{U_J^2}{h} + \frac{h^3}{12G} \left(\frac{\partial P}{r \partial \theta} \right)^2 + \frac{h^3}{12\mu^*} \left(\frac{\partial P}{\partial y} \right)^2 \quad (17)$$

The μ^* and G vary with temperature by the following expression.

$$\mu^* = \mu_{T_0}^* e^{-\beta(T_m - T_0)} \quad (18)$$

$$G = G_{T_0} e^{-\beta(T_m - T_0)} \quad (19)$$

The dimensionless steady-state non-adiabatic energy equation is

$$\left(6\bar{h} - \frac{\bar{h}^3}{\bar{G}} \frac{\partial \bar{p}}{\partial \theta} \right) \frac{\partial \bar{T}_m}{\partial \theta} - \left(\frac{1}{\lambda^2} \frac{\bar{h}^3}{\bar{\mu}^*} \frac{\partial \bar{p}}{\partial y} \right) \frac{\partial \bar{T}_m}{\partial y} = \frac{12\bar{\mu}^*}{\bar{h}} + \frac{\bar{h}^3}{\bar{G}} \left(\frac{\partial \bar{p}}{\partial \theta} \right)^2 + \frac{1}{\lambda^2} \frac{\bar{h}^3}{\bar{\mu}^*} \left(\frac{\partial \bar{p}}{\partial y} \right)^2 + 72 \frac{\bar{k}_f}{\bar{h}} (\bar{T}_j - 2\bar{T}_m + \bar{T}_{Bl}) \quad (20)$$

Assuming that the journal surface temperature is constant under the rotation, it is

$$T_J = \frac{1}{2\pi} \int_0^{2\pi} T_m d\theta \quad (21)$$

The heat flux balance at the interface between fluid film and inner surface of the bearing bush is

$$k_B \frac{\partial T_B}{\partial r} \Big|_{r=Bl} = -k_f \frac{\partial T}{\partial z} \Big|_{z=0} = -k_f \left(\frac{2}{h} (3T_m - T_J - 2T_{Bl}) \right) \quad (22)$$

The heat flux balance at the interface between outer surface of the bush and the atmosphere is

$$k_B \frac{\partial T_B}{\partial r} \Big|_{r=BO} = -h_{conv} (T_{BO} - T_{atm}) \quad (23)$$

4. Load capacity, Friction force

The force due to the hydrodynamic pressure on the journal in the V-H coordinate system is calculated as

$$\left(\bar{F}_V \right)_s = - \int_0^1 \int_0^{2\pi} \bar{p}_s \cos \theta' d\theta' d\bar{y}$$

and

$$\left(\bar{F}_H \right)_s = - \int_0^1 \int_0^{2\pi} \bar{p}_s \sin \theta' d\theta' d\bar{y} \quad (24)$$

The friction force in fluid film is obtained by integrating the shear stress on journal surface. Using equation (13) where $z = h$,

the non-dimensional friction force \bar{F}_{fric} is

$$\bar{F}_{fric} = \int_0^1 \int_0^{2\pi} \frac{\bar{\mu}^*}{h} d\theta' d\bar{y} + \int_0^1 \int_0^{2\pi} \frac{\bar{h}}{2} \frac{\partial \bar{p}}{\partial \theta'} d\theta' d\bar{y} \quad (25)$$

5. Results and Discussions

The numerical results of the performance characteristics of journal bearings lubricated with non-Newtonian lubricants based on Carreau viscosity model were calculated. The length to diameter and the radial clearance ratio of the journal bearing are 1.0 and 2×10^{-5} respectively. The lubricant properties and bearing parameters are shown in table1.

The rheological model for Power-law model and Carreau model of the non-Newtonian lubricant are presented in Figure1. The film pressure and temperature distribution at the bearings mid plane along the circumferential direction of journal bearing for eccentricity ratio equal to 0.6, operated speed at 1,000, 2,000, 5,000, 8,000, and 12,000 rpm as shown in Figure2. The film pressure and temperature decreases when increasing the journal speed. The rate of decrease in film pressure and temperature due to increase speed becomes small at high speed. Both higher pressure and higher temperature are obtained for the Carreau model when compared with Power-law model at 5,000 rpm. At this operating condition, the lubricant becomes Newtonian fluid for the Carreau model as shown in Figure 1. Therefore the

viscosity of the Carreau model lubricant is constant while the viscosity for the Power-law lubricant continues to decrease with the increase of the journal speed.

The effects of speed on the dimensionless load capacity and dimensionless friction force for the journal bearing are shown in Figure 3. The load capacity decreases with increased journal speed, however the load capacity slightly deteriorates at high speed. At 5,000 rpm, the difference between the load capacity calculated using Power-model and the load capacity calculated using Carreau viscosity model are significant for bearing operated at eccentricity ratio more than 0.5. At low speed, the friction force is largely due to the film thickness increase. The friction force is rapidly decreased with increased speed.

Journal radius	0.1 m
Outer radius of bearing-bush	0.15 m
Bearing length	0.2 m
Radius clearance	0.0002 m
Inlet lubricant temperature	20 °C
Lubricant specific heat	1840 J kg ⁻¹ °C ⁻¹
limiting viscosity at low shear rate	0.5 Pa s
limiting viscosity at high shear rate	0.002 Pa s
power-law index , <i>n</i>	0.2
time constant , <i>a</i>	0.0015
Lubricant density	880 kg/m ⁻³
Thermoviscosity coefficient	0.315 °C ⁻¹
Thermal conductivity of lubricant	0.125 W/m-K
Thermal conductivity of bearing	59.0 W/m-K
Heat transfer coefficient	11.0 W/m ² -K

Table 1 : Lubricant properties and bearing parameters

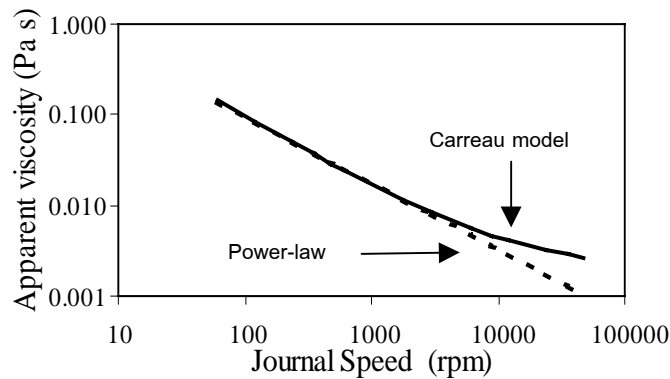


Figure 1 Comparison of Carreau viscosity model with Power-law model

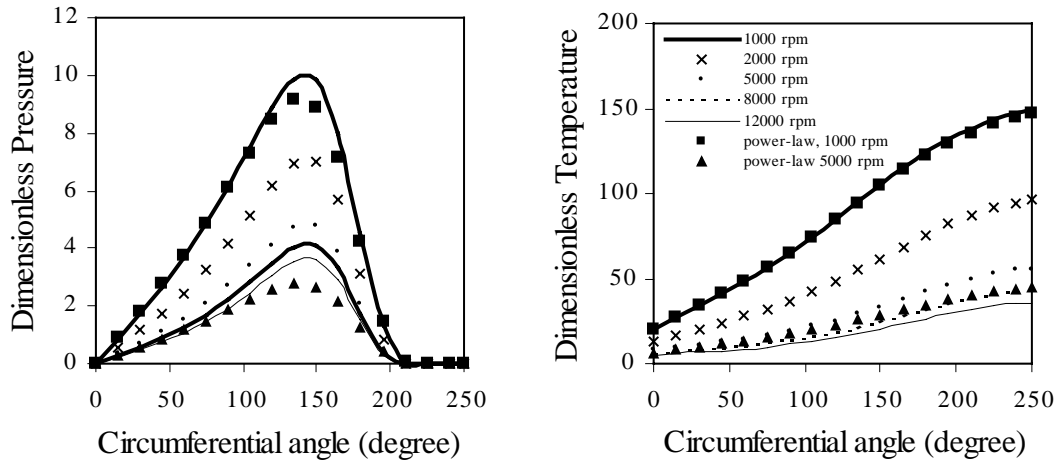


Figure 2 Pressure and temperature distribution along circumferential direction at eccentricity ratio 0.6

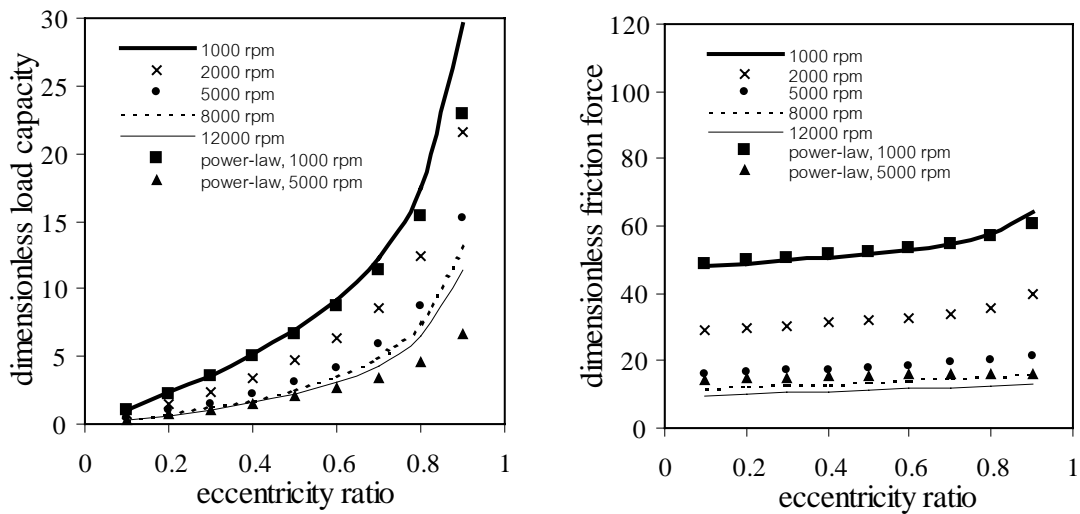


Figure 3 Load capacity and friction force with variation of journal speed

6. Conclusions

The characteristics of journal bearings lubricated with Carreau viscosity model can be concluded as

- 1) The formulation of Reynolds equation and non-adiabatic energy equation of journal bearings with non-Newtonian Carreau viscosity model were presented.
- 2) The effect of speed on pressure and temperature are significant for lubricants with Carreau viscosity model.

- 3) When increasing the journal speed, the viscosity of Pseudo-plastic lubricants rapidly decrease at low speed and slightly decreases at high speed, and significantly effects the reduction of the load capacity and friction force.

Nomenclature

F = Hydrodynamic force (N)
 F_{fric} = Friction force (N)
 T = temperature, °C
 U = tangential velocity of surface, (m/s)
 V = velocity of surface in axial direction, (m/s)
 h = oil film thickness, (m)
 p = fluid film pressure, Pa
 t = time, (s)
 u = velocity of lubricant in circumferential direction, (m/s)
 v = velocity of lubricant in axial direction, (m/s)
 x = coordinate in circumferential direction
 y = coordinate in axial direction
 z = coordinate across fluid film direction
 H = coordinate in horizontal direction

V = coordinate in vertical direction
 δ = a small, nondimensional amplitude parameter for expansion
 ω = angular velocity, (rad/s)
 ε = eccentricity ratio of journal bearing
 μ = apparent viscosity, (Pa s)
 θ = circumferential angle measured from the line of centers, (rad)
 θ' = circumferential angle measured from the negative V-axis, (rad)
 τ_{zx} = component of shear stress in x-direction, (Pa)
 τ_{zy} = component of shear stress in y-direction, (Pa)
 k_f = Thermal conductivity of lubricant
 k_B = Thermal conductivity of bearing
 h_{conv} = Heat transfer coefficient

subscripts

B = bush
 BI = inner bush surface
 BO = outer bush surface
 J = journal

s = steady-state position
 T_0 = at reference temperature
 0 = limiting at low shear rate
 ∞ = limiting at high shear rate

Dimensionless form

$$\bar{F} = \frac{1}{\mu_{ref} U_J L} \left(\frac{c}{r_J} \right)^2 F$$

$$\bar{F}_{fric} = \frac{c}{\mu_{ref} U_J L r_J} F_{fric}$$

$$M = \frac{c \omega^2 m}{F}$$

$$B = \frac{c \omega b}{F}$$

$$K = \frac{ck}{F}$$

$$\bar{p} = \frac{c^2}{\mu_{ref} U_J r_J} (p - p_{atm})$$

$$\bar{T} = \frac{\rho_c c^2}{\mu_{ref} U_J r_J} (T - T_0)$$

$$\bar{\beta} = \frac{\mu_{ref} U_J r_J}{\rho_c c^2} \beta$$

$$\bar{y} = \frac{2}{L} y$$

$$\bar{h} = \frac{h}{c}$$

$$\bar{r} = \frac{r}{r_J}$$

$$\bar{z} = \frac{z}{h}$$

$$\lambda = \frac{L}{D}$$

$$\bar{\mu}^* = \frac{\mu^*}{\mu_{ref}}$$

$$\bar{G} = \frac{G}{\mu_{ref}}$$

$$\bar{\dot{V}} = \frac{\dot{V}}{c \omega_J}$$

$$\bar{\dot{H}} = \frac{\dot{H}}{c \omega_J}$$

$$\bar{\ddot{V}} = \frac{\ddot{V}}{c \omega_J^2}$$

$$\bar{\ddot{H}} = \frac{\ddot{H}}{c \omega_J^2}$$

$$\bar{t} = \omega t$$

$$\bar{k}_B = \frac{k_B}{\rho_c \nu \omega c r_J}$$

$$\bar{k}_f = \frac{k_f}{\rho_c \nu \omega c^2}$$

$$\bar{h}_{conv} = \frac{h_{conv}}{\rho_c \nu \omega c}$$

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