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Optimum Synthesis of Steering Mechanism

Suwin Sleesongsom[1],*

[1] Department of Mechanical Engineering, Faculty of Engineering, Chiangrai College, Chiangrai City, Thailand
*suwin sleesongsom: suwins2000@yahoo.com, 089-4275255, 053-170331-3

Abstract

At the present, human's life needs to have more facilities and safety. In daily life, we regularly use vehicles. When we talk about the safety in driving cars, there is nothing more important than the steering systems, which work properly in order to maintain themselves to be in safety condition. A steering system is the mechanism to control car to the direction that the driver wants. From these reasons, that became the objective of this research that is required to synthesize the steering mechanism, which has minimum steering error against the Ackerman's theory. The optimization problem is a single objective optimization of the steering error. The optimization constraints are assigned that based on the limitations of steering mechanism. The design variables are tie rod (L_t), steering arm (L_a), and the distance between the front wheel kingpin and rack (H). The optimizer is used to find the optimum result is the optimization toolbox in Matlab, which is called fmincon. The study found that the optimum design variables are L_a = 300 mm, L_t = 300 mm, and H = 229.3795 mm, which had minimum of the maximum steering error when comparing with Ackerman's theory is 0.0836°. From the steering error is obtained from the designing, it showed that the proposed of designing method is an efficient method to reduce the steering error, which can reduced the skidding and wear of the tires.

***Keywords**:* Rack-and-pinion steering linkage; Steering error; single-objective optimization

1. Introduction

The rack-and-pinion steering linkages are used in small cars. The advantage of this steering linkage is simple to construct, economical to manufacture, and compact and easy to operate. It consists of rack-and-pinion, two steering arms, and two tie rods. The linkage has two types are central take-off (CTO) and side take-off (STO), which can be divided as trailing or leading. For instant, in central take-off type, the tie rods and the rack are connected at the middle of the rack, while in side take-off (STO) type, the tie rods and rack are connected at the rack ends [1]. It has been found, to reduce the skidding and wear of the tires, the steering mechanism of a car must follow the Ackermann principal. This principal states that all the axial lines of the wheels should meet at a point when a car turning with low speed and free from lateral inertia forces, which is called the instantaneous center of turning as shown in Fig. 1 [1]. The outer wheel angle according to Ackernann principal, θ_{OA} is related to the inner wheel angle, θ_I as follow

$$\begin{aligned}\theta_{OA}(\theta_I) &= \tan^{-1}\frac{1}{\cot\theta_I+\dfrac{W_t}{W_b}} \\ &= \tan^{-1}\frac{1}{\cot\theta_I+\dfrac{1}{w_b}}\end{aligned} \tag{1}$$

where w_b = W_b/W_t is the normalized expression of the wheel base, W_b, with respect to wheel track, W_t.

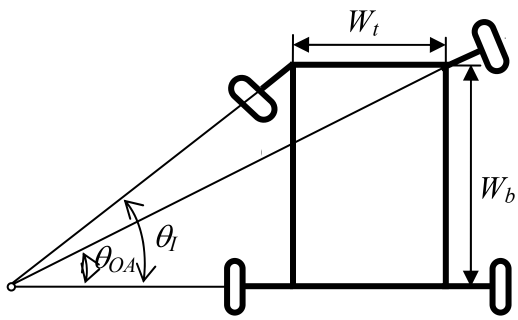

Fig. 1. Akermann principal.

Unfortunately, there is no mechanism that can fulfill the Ackermann principal at every turning radius, except six-bar mechanism [2]. Later, there is an attention of many researchers to synthesize the six-bar steering linkages, which can satisfy the Ackermann principal for some orientations. Optimization of steering error between actual steering linkage with rack-and-pinion and Ackermann principal has been studied by many researchers [2-8]. From my review of literatures found that the optimization problems have been only studied to minimize the steering error [2-9] and link-length sensitivity [1, 7] with or without the normalized link length variables. The most optimization problems are a single objective, except the work by Hanzaki et al. [1], they used weight sum to combine sensitivities to be an objective. The weight sum is a method to posteriori satisfaction of many objectives by a designer, which is a multi-objective optimization technique. Furthermore, the optimizers have been used to minimize steering error is the method based on gradient, except the work by Peñuñuri et al. [2], they used the method based on evolutionary method, which called differential evolution (DE).

This paper is intended as an extension of above literature. The purpose of this research is to minimize the maximum steering error, while the design variables are normalized linkages length. The design problem is solved by method based on the sequential quadratic programming, which is called fmincon. This optimizer is an optimization tool in commercial software MATLAB.

The rest of this paper is organized as follows. Section 2 derives the steering error of six-bar linkage. The optimization problem is proposed in section 3. The optimizer is proposed in section 4. A numerical experiment and the design results are given in section 5. The conclusions of this study are finally drawn in section 6.

2. Kinematic analysis

The steering error of six-bar linkage is proposed in this section. The model of six-bar linkage is shown in Fig. 2, which has been proposed by Hanzaki et al. [1]. The mechanism is CTO configuration, which is leading type. This mechanism is composed of six links and seven joints that are denoted by (1),…,(6) and 1,…,7, respectively. By Gruebler's equation, the mobility or degree of freedom of the links is one.

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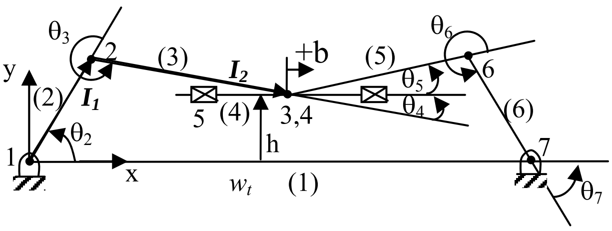

Fig. 2. Six-bar planar rack-and-pinion steering linkage for the CTO and STO configuration.

The six-bar planar rack-and-pinion steering linkage composes of two steering arms (2, 6) with length L_a, two tie rods (3, 5) with length L_t and one rack (4) with lack displacement b.

From Fig. 2, H denotes the distance from the front wheel axle to the steering rack axis. To normalized link length, the linkage lengths except rack length are divided by (W_t-L_r) when L_r is rack length, while the wheel base and wheel track, i.e., W_b and W_t, are divided by W_t. Such that, the normalized parameters, L_a, L_t, H, W_b, and W_t are denoted as l_a, l_t, h, w_b, and w_t, respectively.

To derive the steering error, the actual angle made by the outer front wheel during turning left, θ_O, must be derived first to compare with the correct angle, θ_{OA}, for the same wheel based on the Ackermann principle given by Eq. (1) at every inner wheel rotation, θ_I . From Fig. 2,

$$\mathbf{I}_1+\mathbf{I}_2=\left(\frac{w_t}{2}+b\right)\mathbf{i}+h\mathbf{j} \tag{2}$$

where $\mathbf{I}_1$,and $\mathbf{I}_2$ are the addition vector to the center of rack, w_t is the wheel track, b is the rack displacement, h is the distance from the front wheel axle to the rack axis, and **i** and **j** are the unit vectors along x and y, respectively. Eq. (2) can be written in terms of its scalar components as

$$l_a\cos\theta_2+l_t\cos\theta_{23}=\frac{w_t}{2}+b \tag{3}$$

$$l_a\sin\theta_2+l_t\sin\theta_{23}=h \tag{4}$$

where $\theta_{23}=\theta_2+\theta_3$.

When a car goes straight ahead before turning, the initial angle of the left, θ_{20} are equal to the initial angle on the right, θ_{70}. The derivation of the left steering angle starts with rearrange Eqs. (3, 4) as follow

$$l_t\cos\theta_{23}=\frac{w_t}{2}+b-l_a\cos\theta_2 \tag{5}$$

$$l_t\sin\theta_{23}=h-l_a\sin\theta_2 \tag{6}$$

Square and summing Eq. (5, 6):

$$l_t^2(\sin^2\theta_{23}+\cos^2\theta_{23})=\left(\frac{w_t}{2}+b-l_a\cos\theta_2\right)^2+\left(h-l_a\sin\theta_2\right)^2 \tag{7}$$

Use the trigonometric relation, $\sin^2\theta_{23}+\cos^2\theta_{23}=1$, Eq. (7) becomes

$$A\cos\theta_2+B\sin\theta_2=k_2 \tag{8}$$

where $A=\left(w_t+2b\right)$, $B=2h$,

$k_1=-l_t^2+l_a^2+h^2$ and

$k_2=\left[k_1+\left(\frac{w_t}{2}+b\right)^2\right]/l_a$.

To solve Eq. (8), Divide Eq. (8) by $\sqrt{A^2+B^2}$ yields

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$$\sin\alpha\cos\theta_2 + \cos\alpha\sin\theta_2 = \frac{k_2}{\sqrt{A^2+B^2}} \tag{9}$$

Use the trigonometric relation,

$\sin(\alpha+\theta_2) = \sin\alpha\cos\theta_2 + \cos\alpha\sin\theta_2$,

Eq. (9) becomes

$$\theta_2 = \sin^{-1}\left(\frac{k_2}{\sqrt{A^2+B^2}}\right) - \alpha + \frac{\pi}{2}. \tag{10}$$

Likewise, for the right steering angle, θ_7 can be obtained as

$$\theta_7 = -\sin^{-1}\left(\frac{k_3}{\sqrt{A^2+B^2}}\right) - \alpha + \frac{\pi}{2} \tag{11}$$

where $k_3 = \left[k_1 + (\frac{w_t}{2} - b)^2\right] / l_a$.

In practical, the range of the inner wheel rotation of the small cars is 0-40°. For the leading configuration, $\theta_2 = \theta_{20} + \theta_I$ when turning left. To evaluate angle θ_7 in an arbitrary position of the mechanism, the expression of the rack displacement is calculated by rearrangement Eq. (7) to obtain:

$$b = l_a\cos\theta_2 - \frac{w_t}{2} \pm \sqrt{\Delta} \tag{12}$$

where $\Delta = l_a^2\cos^2\theta_2 - k_1 + 2l_a h\sin\theta_2$. In Eq. (12), positive sign is used. Furthermore, for straight ahead configuration θ_{20}, is equal to θ_{70}, which is found by Eqs. (10, 11) by set the lack displacement b = 0. The actual angle made by the outer front wheel during turning left, θ_O, is $\theta_O = \theta_{70} - \theta_7$.

2.1 Steering error

To protect the wheel from wear and skidding, the steering linkages must accord with Akermann principal, which can be accomplish by minimizing the steering error, $\delta\theta_O$:

$$\delta\theta_O = |\theta_O - \theta_{OA}| \tag{13}$$

where θ_O is the actual angle made by the outer front wheel during turning left and θ_{OA} is the correct angle for the same wheel based on the Ackermann principal, which is given by Eq. (1).

3. Optimization Problem

From the previous section, the steering error is derided. The optimization problem of the steering error is performed in this section.

Minimize $f_1 = \max|\delta\theta_O|$ (14)

Subject to $\Delta \geq 0$;

$|b_i| \geq |b_{i-1}|$ for i = 1,2, … ,n

$0.1 \leq b_i \leq 0.14$ m

$0.01 \leq H \leq 0.3$ m

$0.01 \leq L_a, L_t \leq 0.3$ m

where $\delta\theta_O$ is the steering error, *b* is the rack displacement, *H* is the distance from the front wheel axle to the steering rack axis, L_a is the link length of steering arm, and L_t is the link length of tie rod. The limiting constraints are assigned based on work of Hanzaki et al. [1] and manufacturer data.

4. Optimizer

The optimizer is used for solving the optimization in this research is the optimization toolbox in the well known commercial software MATLAB, which name is fmincon. The toolbox is based on the sequential quadratic programming (SQP), which is properly for solving the nonlinear constraint optimization. The flow diagram of this algorithm is beyond the scope of this paper.

5. Numerical Experiment

The purpose of this research is to find the link length of a rack-and-pinion steering linkage, which is minimized maximum steering error. The kinematics optimization for the rack-and-pinion steering linkage that based on the concepts outlined above is illustrated with the help of an STO trailing linkage used in a commercial small car [1], which has its' dimension as shown in Table 1. The actual values are approximated from manufacturer's data. The real dimensions of the small car appearing in the 1st column in Table 1 is first normalized and shown in the 2nd column.

The validation of steering error using for the program code in this work is based upon the rack-and-pinion steering linkage, which has been previously reported by Hanzaki [1]. In the literature, the maximum steering error of the STO trailing linkage is 3.9, which has its' dimension as shown in table 1 and its' initial dimensions such as L_a, L_t, and H are 110 mm, 256 mm, and 177 mm, respectively. From the authors' computer code, the steering error is 3.8.

To solve this optimization problem, the fmincon is applied to solve this design problem, where the maximum total number of iterations is set to be 100, and the tolerance of the function, constraint, and design variables is 1×10^{-6}. The procedure is terminated when reaching the maximum iteration number or close to the tolerance.

Table 1 Dimensions of a small car

	Initial dimension (mm)	Normalized dimension
Wheel base, W_b	2175	1.7901
Wheel track, W_t	1215	1
Rack length, L_r	678	

The optimization result is shown in Table 2. Figure 3 displays the search history as the plot of iteration number against the steering error. The search history only shows a few iteration numbers that the solution can converse to an optimum solution, which is mean that the optimizer is height performance to find the solution. The performance of this optimizer as a result to the steering error of the present solution point is lower than the previous work by Hanzaki et al. [1].

Table 2 The optimum result

	Kinematic optimum dimensions
Steering arm length, L_a	300
Tie rod length, L_t	300
Distance from rack axis to wheel axis, H	229.3795
Max. error, $\delta\theta^{\circ}$	0.0836

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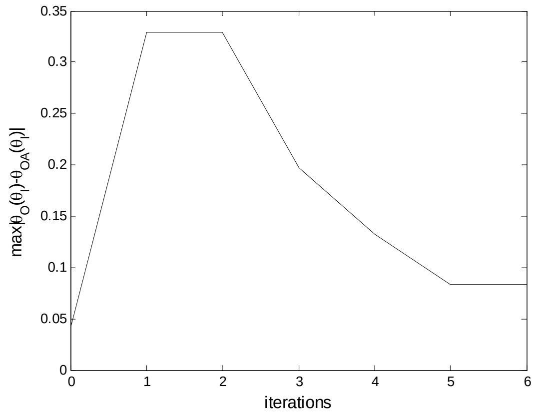

Fig. 3 Search history of the steering linkage.

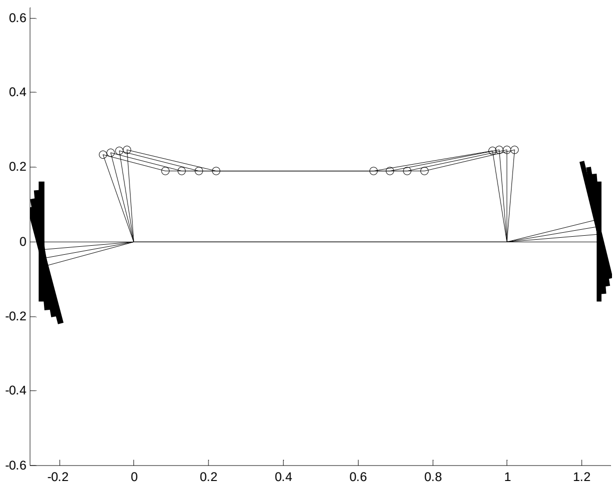

Fig.4 Linkage layouts of the optimum steering linkages.

To study the performance of the new design solution, a turning left is studied. The steering error need to be examined at various turning left angle. The turning left angle, θ_I is in the range of 0-40°, when the increasing angle is 5°. The steering error of the steering linkage at the various turning angles are given in Fig. 4 and in Table 3. Furthermore, the result shows that the maximum inner wheel angle of this steering linkage is not more than 12.6°, which is result from rack stroke constraint.

Table 3 The steering error at various turning angle.

θ_I(°)	θ_o(°)	θ_{OA}(°)	$\delta\theta_o$
0	0	0	0
5	4.8663	4.7681	0.0982
10	9.4847	9.1191	0.3656
15	13.8893	13.1194	0.7699

6. Conclusions

This paper proposed that the method to synthesize the steering linkages, which can minimum the maximum steering error. The kinematic optimization of the steering linkage is carried out the using three homogenous design parameters are steering arm and tie rod length, and the distance from the front wheel axis to the rack axis. All parameters have the unit of length. The proposing method is high performance to find the optimum solution. The study found that the optimum design variables are L_a = 300 mm, L_t = 300 mm, and H = 229.3795 mm, which had minimum the maximum steering error when comparing with Ackerman's theory is 0.0836°. From the steering error is obtained from the designing, it showed that the proposed of designing method is an efficient method to reduce the steering error, which can reduced the skidding and wear of the tires.

Our future work can apply this designing concept to a real steering linkage, which is considered both steering error and turning radius.

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7. Acknowledgement

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