Improving Robustness of Input Shaping Technique with Model Reference Iterative Learning Control

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Abstract

Input shaping technique designs a sequence of impulses so that the system's impulse responses cancel out, resulting in zero residual vibration. The technique is a feed-forward technique that relies on an accurate knowledge of the system's natural frequencies and damping ratios. If this information is inaccurate or the system changes with time, the vibration reduction performance deteriorates. In this paper, for the first time, a novel closed-loop system to improve robustness of the input shaping technique is proposed. An iterative learning control is placed in the loop to match the closed-loop mapping to a reference model. The input shaper can then be designed on the natural frequencies and damping ratios of the reference model, which are accurate. This proposed technique is very effective especially for a system having repeat reference, repeat disturbance, and repeat initial conditions. A simulation example demonstrates its effectiveness.

***Keywords**:* Input shaping; Iterative learning control; Vibration reduction; Model reference.

1. Introduction

Input shaping technique was proposed by [1], based on the Posicast control idea of [2]. The technique computes optimal magnitudes and time locations of impulses in an impulse sequence. This sequence is implemented as an FIR filter whose frequency response magnitude is low around the system's natural frequencies thus avoiding resonances.

In input shaping technique, the amplitudes and time locations of the impulses are designed from the knowledge of the system's natural frequency and damping ratio. However, actual system can be different from the model system for various reasons, including imperfect system identification, time-varying system, and configuration-dependent system. This uncertainty deteriorates the performance of the input shaping in attenuating the residual vibration.

To make the input shaping technique more robust to this uncertainty, more impulses have been added as in [3] – [5] with a disadvantage of slower reference input, and several adaptive input shaping schemes [6] – [8] have been proposed with problems of lack of persistent excitation of input and divergence of parameter estimates.

Some researchers have used model reference idea with input shaping technique.

Control system was used to match the closed-loop system with the reference model, whose natural frequency and damping ratio were used in designing the input shaper. A sliding mode controller was used in [9], a PI controller in [10], and a direct model reference adaptive control in [11] for reference model matching. Some disadvantages of the proposed techniques above include complexity, global convergence, heuristic design, lack of persistent excitation of input, and global instability.

Iterative learning control (ILC) is a performance-enhancing feed-forward control for systems that repeat the same trajectory or task. ILC can be viewed as an adaptive control system that learns from previous iteration. Advantages include being a simple and model-independent algorithm and there is no need for the input persistent excitation. A survey of ILC is given in [12].

In this paper, for the first time, the ILC is proposed with input shaping in the model reference setting. A frequency-domain ILC is used with a PID feedback control to match the closed-loop system with a reference model. Via a simulation example, with an uncertain plant, the performance of the input shaper was shown to improve to that with the nominal plant after several iterations.

The paper is organized in this way. A brief review of the input shaping technique is given in Section 2. Section 3 contains the proposed model reference iterative learning control (MRILC) with input shaping. Simulation example is given in Section 4 and conclusions in Section 5.

2. Input Shaping Technique

This technique computes amplitudes and time locations of an impulse sequence, so that when it is applied to a linear system, its impulse responses cancel out resulting in zero residual vibration.

It was shown in [13] that the ratio between the n-impulse response amplitude at time $t \geq t_n$ and the single-impulse response amplitude at time $t \geq t_1$ is given by

$$V(\omega_n,\zeta) = e^{-\zeta\omega_n t_n}\sqrt{\left[C(\omega_n,\zeta)\right]^2 + \left[S(\omega_n,\zeta)\right]^2},$$

$$C(\omega_n,\zeta) = \sum_{i=1}^{n} \hat{F}_i e^{\zeta\omega_n t_i}\cos\left(\omega_n\sqrt{1-\zeta^2}\,t_i\right),$$

$$S(\omega_n,\zeta) = \sum_{i=1}^{n} \hat{F}_i e^{\zeta\omega_n t_i}\sin\left(\omega_n\sqrt{1-\zeta^2}\,t_i\right),$$

where V is the so-called *percentage vibration*, normally used in the literature to quantify the residual vibration, ω_n is the natural frequency of the applied linear system, ζ is its damping ratio, t_i is the time the i^{th} impulse is applied and $\hat{F}_i$ is the i^{th} impulse's amplitude.

The amplitudes $\hat{F}_i$ and time locations t_i of the impulse sequence are computed by solving the following equations:

$$V(\omega_n,\zeta) = 0, \quad (1)$$

$$\frac{\partial V(\omega_n,\zeta)}{\partial \omega_n} = 0, \quad (2)$$

$$\sum_{i=1}^{n} \hat{F}_i = 1, \quad (3)$$

$$t_1 = 0, \quad (4)$$

which requires the knowledge of ω_n and ζ.

Eq. (1) ensures the residual vibration at time $t \geq t_n$ is zero. It leads to $C(\omega_n,\zeta) = 0$ and

$S(\omega_n,\zeta)=0,$ which can be solved for two unknowns.

Eq. (2) reduces the sensitivity of V to the variations in the natural frequency and damping ratio. Note that $\partial V(\omega_n,\zeta)/\partial\zeta=0$ is equivalent to Eq. (2), and both lead to $\partial C(\omega_n,\zeta)/\partial\omega_n=0$ and $\partial S(\omega_n,\zeta)/\partial\omega_n=0,$ which can be solved for two unknowns. Their higher-order derivatives can also be used to provide even higher-level of robustness.

Eq. (3) is such that the shaped reference has the same final value as that of the original reference. It can be solved for one unknown. Eq. (4) marks the time origin of the first impulse, which is already an unknown.

Eqs. (1) - (4) are used to solve six unknowns, which are the amplitudes and time locations of three impulses:

$$t_1=0,\ \hat{F}_1=\frac{1}{1+2K+K^2}, \tag{5}$$

$$t_2=\frac{\pi}{\omega_n\sqrt{1-\zeta^2}},\ \hat{F}_2=\frac{2K}{1+2K+K^2}, \tag{6}$$

$$t_3=\frac{2\pi}{\omega_n\sqrt{1-\zeta^2}},\ \hat{F}_3=\frac{K^2}{1+2K+K^2}, \tag{7}$$

$$K=e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}. \tag{8}$$

This three-impulse input shaper is known in the literature as zero-derivative-vibration (ZVD) shaper.

The resulting sequence of impulses can be put as coefficients b_k in the unit impulse response

$$h[n]=\sum_{k=0}^{m}b_k\delta[n-k],$$

where $\delta[n-k]$ is the discrete-time unit impulse, and can be implemented as a causal discrete-time FIR filter as shown in Fig. 1.

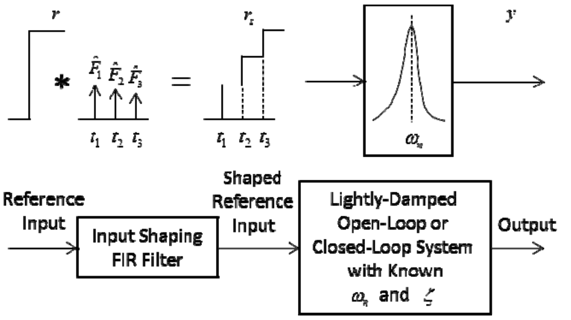

Fig. 1 Implementation of input shaping technique.

3. Model Reference Iterative Learning Control

The proposed system is shown in Fig. 2, where $G_m(s)$ is the reference model whose natural frequency and damping ratio are known precisely and are used to design the input shaping filter $IS(z)$, $C(s)$ is a feedback controller, $P(s)$ is the lightly-damped actual plant, the “ILC” and “Memory” blocks represent the iterative learning controller. $r, r_s, u, u_1, u_2, y, y_m$, and e are the repeating reference, shaped reference, total control effort, feedback control effort, iterative control effort, actual output, reference model output, and output error, respectively. j denotes the j^{th} iteration of the iterative control.

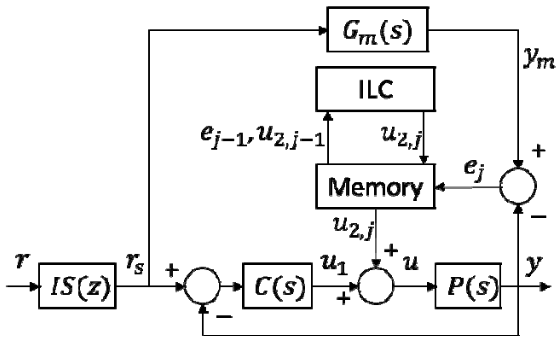

Fig. 2 Input shaping with model reference iterative learning control.

The ILC's objective is to reduce the output error $e = y_m - y$ so that the actual closed-loop system behaves like the reference model to obtain the best vibration reduction performance from the input shaper.

A general ILC algorithm is of the form

$$u_{2,j+1}(k) = Q(z)\left[u_{2,j}(k) + L(z)e_j(k)\right], \quad (9)$$

where $Q(z)$ and $L(z)$ are two filters to be designed.

Normally, $Q(z)$ is a band-pass filter that selects a range of frequencies where learning occurs. $L(z)$ can be any type of filter but most common types are P-type $(L(z) = \gamma z)$ and PD-type $(L(z) = \gamma z + \lambda(z-1))$, where γ and λ are two positive design constants.

For example, the P-type ILC algorithm (9) with $Q(z) = 1$ is given by

$$u_{2,j+1}(k) = u_{2,j}(k) + \gamma e_j(k+1), \quad (10)$$

which can be thought of as an integrator in the iteration domain. Also, the error is measured one time step before the control effort is computed.

For our problem, a condition that guarantees the convergence of the ILC control effort u_2 can be found as follows:

$$\begin{aligned} e_j &= y_m - y_j \\ &= G_m r_s - y_j \\ &= \left(G_m - \frac{CP}{1+CP}\right) r_s - \frac{P}{1+CP} u_{2,j} \\ &= e_0 - H u_{2,j}, \end{aligned}$$

$$\begin{aligned} u_{2,j+1}(k) &= Q(z)\left[u_{2,j}(k) + L(z)e_j(k)\right] \\ &= Q(z)\left(1 - L(z)H(z)\right)u_{2,j} \\ &\quad + Q(z)L(z)e_0. \end{aligned}$$

Therefore, a contractive mapping from $u_{2,j}$ to $u_{2,j+1}$ is achieved if

$$\left\| Q(z)\left(1 - L(z)H(z)\right) \right\|_\infty < 1. \quad (11)$$

4. Simulation Result

Consider a mass-spring-damper system as shown in Fig. 3. In general, the system represents two entities, connected via a flexible part. The driving one has an absolute position of x_1, and the driven one has x_2. This system was used as benchmark in [14] to illustrate the effectiveness of an input shaping scheme.

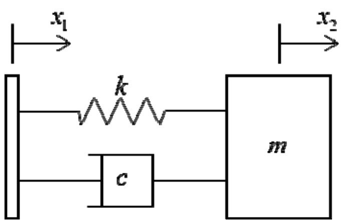

Fig. 3 An m-c-k system as an illustrative example.

The system above has a transfer function relating x_1 to x_2 as

$$\frac{x_2}{x_1} = \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \quad (12)$$

where ω_n and ζ are the system natural frequency and damping ratio.

For simulation purpose, let the nominal ω_n be 10 rad/s and ζ be 0.3, and let the parameter variations be $\pm 50\%$ from their nominal values.

Eq. (12) represents the plant $P(s)$ in Fig. 2. The input shaping with model reference iterative learning control in Fig. 2 will be applied to this system.

Fig. 4 shows root locus plot of the nominal plant with $C(s)=1$. The nine shaded squares belong to nine possible plants, whose parameters are in the sets $\omega_n \in \{5, 10, 15\}$ and $\zeta \in \{0.15, 0.30, 0.45\}$. Even though the closed-loop system, in all cases, is stable, the settling time appears to be slow (0.4 seconds for the nominal case) and the steady-state tracking error is not zero because the open-loop system is of type 0.

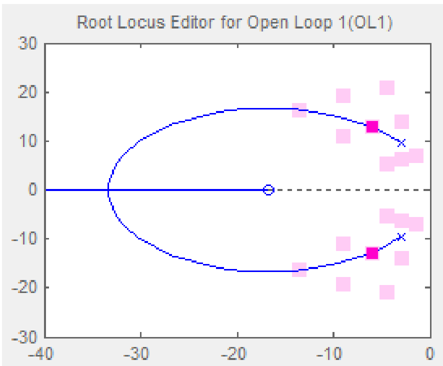

Fig. 4 Root locus with $C(s)=1$.

To improve transient performance, a PID controller with a derivative filter was designed using the Ziegler-Nichols step response method as

$$C(s)=\frac{177.33\left(s^2+319.1s+2.665\times10^4\right)}{s(s+3424)}. \quad (13)$$

Fig. 5 shows the new root locus plot. The settling time for the nominal case is around 0.06 seconds and the steady-state tracking error is zero.

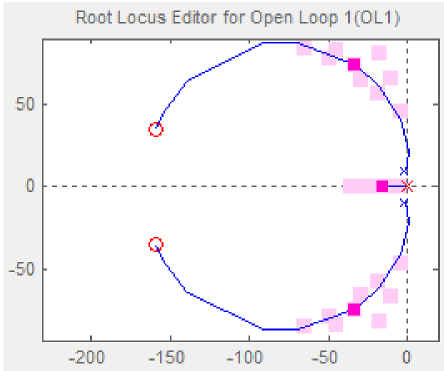

Fig. 5 Root locus with a PID controller.

The reference model is the closed-loop system

$$G_m(s)=C\bar{P}/\left(1+C\bar{P}\right),$$

where $\bar{P}$ is the nominal plant (12) with nominal values of ω_n and ζ and C is the controller (13). $G_m(s)$ has one lightly damped mode with $\zeta=0.4154$ and $\omega_n=81.7\ rad/s$. These values are used in designing the ZVD input shaper $IS(s)$, which is given by (5) – (8).

The P-type ILC (10) is used with $\gamma=10$. The magnitude of the transfer function

$$1-\frac{\gamma zP}{1+CP}$$

is plotted in Fig. 6 for all nine possible plants. It can be seen that the condition (11) is met and the ILC control effort will convert with the designed parameters.

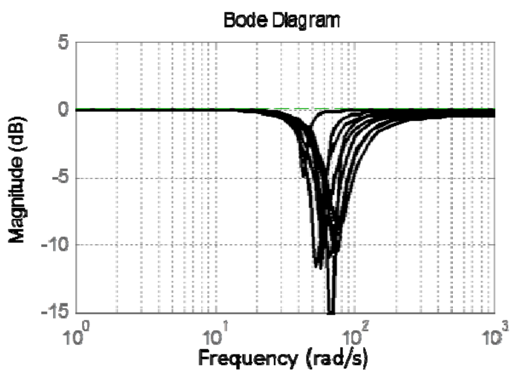

Fig. 6 Magnitude plot of $1-\gamma zP/(1+CP)$.

Consider first the closed-loop system in Fig. 2 when the model reference ILC is turned off. When the input shaper is not used, the output y from the nominal plant vibrates near the set-point as shown in Fig. 7.

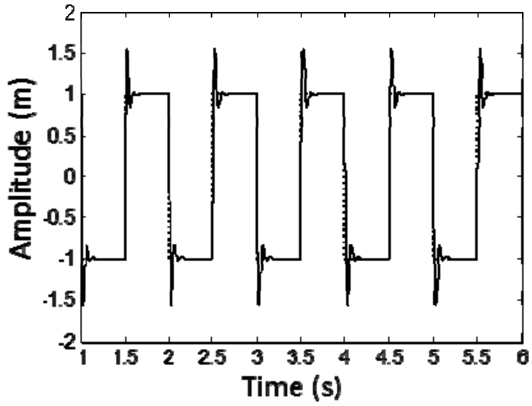

Fig. 7 Nominal plant without the input shaper: (dash) reference r, (solid) output y.

When the input shaper is turned on, residual vibration disappears as shown in Fig. 8.

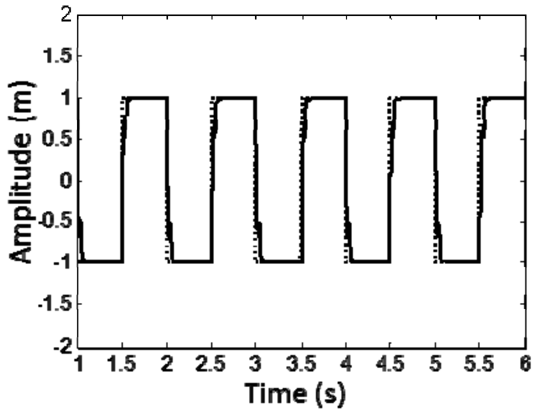

Fig. 8 Nominal plant with the input shaper: (dash) reference r, (solid) output y.

However, when the plant is not at its nominal value, the input shaper's performance in reducing the residual vibration deteriorates as shown in Fig. 9. The perturbed plant (12) has $\omega_n = 5\, rad/s$ and $\zeta = 0.15$.

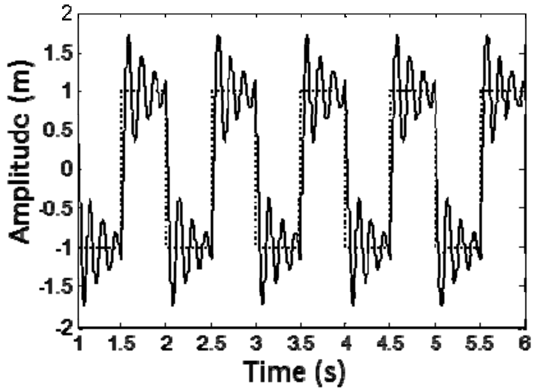

Fig. 9 Perturbed plant with the input shaper: (dash) reference r, (solid) output y.

When the model reference ILC is turned on, its control effort u_2 is designed to reduce the model output error $e = y_m - y$ over each iteration. As time passes, the closed-loop system behaves closer to the reference model; therefore, the input shaper, which was designed from the reference model, performs better in reducing the residual vibration. This can be seen from the result in Fig. 10, which presents the case of Fig. 9 but when the MRILC is turned on.

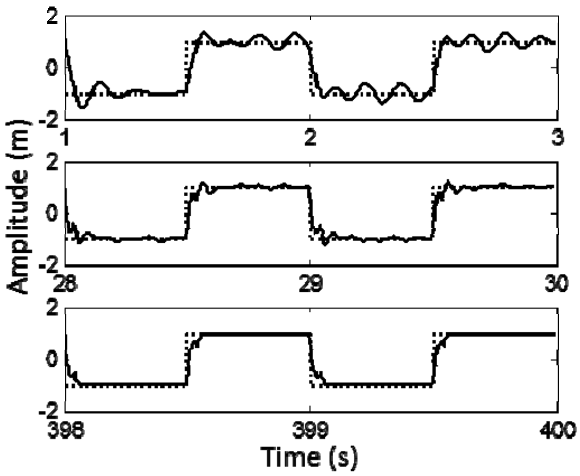

Fig. 10 Perturbed plant with the input shaper and with MRILC: (dash) reference r, (solid) output y.

Fig. 11 shows the root-mean-square values, taken over each iteration, of the model error $e = y_m - y$, the feedback control effort u_1, and the ILC control effort u_2. The error is reduced to almost zero during the 400th iteration. The feedback control effort is replaced by the ILC control effort especially at the beginning (first 25 iterations). The ILC control effort then converts to a steady-state value.

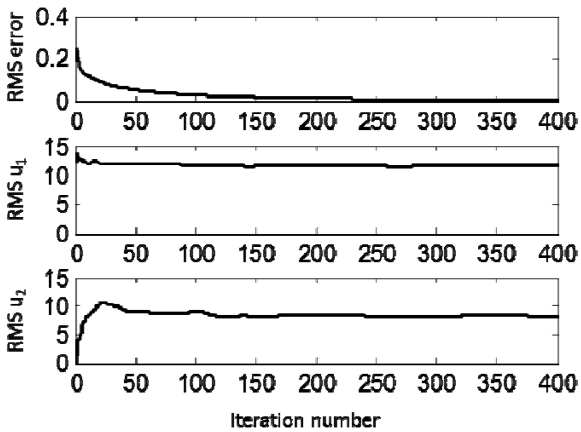

Fig. 11 (Top) Root-mean-square (RMS) value of the error $e = y_m - y$. (Middle) RMS value of the feedback control effort u_1. (Bottom) RMS value of the ILC control effort u_2.

Fig. 12 shows the error e, the feedback control effort u_1, and the ILC control effort u_2 during the 1st, 10th, 100th, and 400th iterations. As the iteration number increases, the model error reduces to zero, and the control effort changes from using feedback to using the feed-forward ILC control.

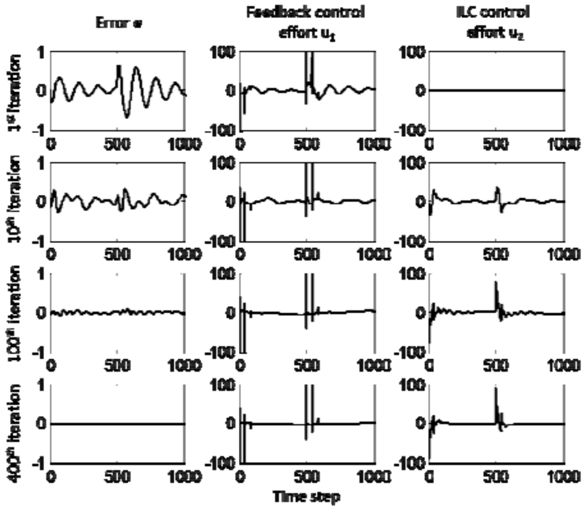

Fig. 12 The error e, the feedback control effort u_1, and the ILC control effort u_2 during the 1st, 10th, 100th, and 400th iterations.

5. Conclusions

A novel closed-loop system to improve robustness of the input shaping technique against uncertain knowledge of the plant's natural frequency and damping ratio is proposed.

The system uses the iterative learning control to match the closed-loop system with a reference model whose parameters were used in designing the input shaper. As a result, the input shaper can recover its nominal performance even under uncertain plant.

More researches are called on time-domain ILC as well as to implement this proposed system to an actual lightly damped system.

6. References

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