The 6th TSME International Conference on Mechanical Engineering
16-18 December 2015

AME001

Unscented Kalman Filtering Implementation for MEMS Accelerometer Tilt Sensing

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Abstract

In this paper, the Unscented Kalman Filter (UKF) is introduced in application of the tilt angle state estimator for MEMS accelerometer tilt sensing. The first, the accelerometer tilt sensing model is development, then the unscented kalman filter with theoretical of minimal complexity of mathematical model and their parameter were derived. Two experiments have been done. First, testing in tilt angle is constant. And the other hand, the testing platform varies angle on level plane and then kept constant angle. Experimental results have shown that the proposed method have good performance while estimating tilt angle.

Keywords*:* Kalman Filter (KF), Unscented Kalman Filter (UKF), Accelerometer Tilt Sensing

1. Introduction

Tilt sensing has been becoming a requirement in a variety of applications, such as robotic attitude control [1, 2], two-wheeled balanced vehicle attitude monitor [3, 4], attitude navigation system [5, 6], human movement monitor [7, 8], and underground drilling [9, 10]. These applications mainly depend on be roughly divided into positioning, aligning, leveling, navigation and orientation. In a general case, a tilt sensing is usually expressed as two component angles including roll and pitch angle [11–13]. For tilt angle measurement, following sensors are mostly used such as inclinometer. This sensor output is proportional to the tilt angle with respect to the field of gravity, called accelerometer.

With small size, low power consumption, low cost, and high reliability, micro-electro mechanical systems (MEMS) accelerometers can be adopted tilt sensing. It has received much attention in recent years. On the other hand, it has some disadvantage, such as low resolution, a high level of noise, worse bias stability, etc., limiting its usage in navigation systems. So, the estimator will be done. Because of accelerometer tilt sensing model is nonlinearity model, the estimator that is the Unscented Kalman Filter (UKF) is used to estimate tilt angle.

In this paper, the Unscented Kalman Filter is introduced once again, this time, in application of the tilt angle state estimator for MEMS accelerometer tilt sensing. For this purpose, theoretical of minimal complexity of mathematical model was derived.
This paper is organized as follows: section 2 presents the mathematical model of accelerometer tilt sensing under consideration and section 3 describes the theoretical and parameter of Unscented Kalman Filter implementation. The Experiment setup and the results and analyze filter performance are discussed in section 4 and section 5. And the last section, conclusion of this work is discussed.

2. Accelerometer Tilt Sensing Model

An accelerometer is used to measure specific forces such as the earth gravity. In figure 2, the sensor cluster compose of 2 accelerometers (A_x and A_y) placed absolutely perpendicular to each other.

In figure 2.1(a), the sensor is mounted completely level in the horizon plane with respect to the earth surface. The x- and y-axis of accelerometer is not affect any gravity. So, the gravity is 0g.

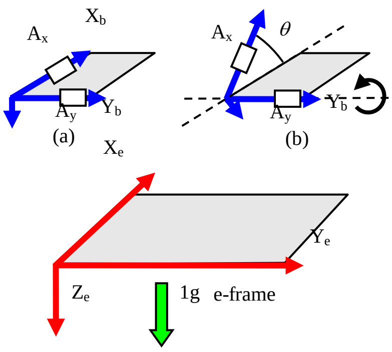

Fig 2.1 The sensor cluster placed absolutely perpendicular to each other

In Figure 2.1(b), if sensor cluster is pitched, the accelerometer A_x will know in advance the gravity to be $-g\sin\phi$, but A_y will still perceive the gravity to be 0g. So, calculating pitch angle (rotation around y-axis) from measured specific forces as follows:

$$\phi = \sin^{-1}(-A_x) \quad (1)$$

From figure 2, considering Euler angles and direction cosine matrix transformation [4]

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = R_x(\theta)R_y(\phi)R_x(\psi)\begin{pmatrix} A'_x \\ A'_y \\ A'_z \end{pmatrix} \quad (2)$$

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} -\sin\phi \\ \sin\theta\cos\phi \\ \cos\theta\cos\phi \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (3)$$

From (3), the roll angle (rotation around x-axis) and the pitch angle (rotation around y-axis) can be determined as follow:

$$A_x = -\sin\phi \quad (4)$$

$$A_y = \sin\theta\cos\phi \quad (5)$$

But, measured accelerations can be corrupted by several disturbances that are random measurement noises. So,

Measurements = Actual Value+Noise (6)

From (4) and (5), having

$$\tilde{A}_x = -\sin\phi + \tilde{v}_x \quad (7)$$

$$\tilde{A}_y = \sin\theta\cos\phi + \tilde{w}_y \quad (8)$$

where $\tilde{A}_x$ and $\tilde{A}_y$ are actually measured accelerations measured from A_x and A_y, respectively. $\tilde{v}_x$ and $\tilde{w}_y$ are random measurement noises of A_x and A_y, respectively.

Eqn.(7) and (8) are system model that describe the relationship between the tilt angles and the output specific forces measured by accelerometers.

3. Unscented Kalman Filter (UKF)

3.1 UKF Model Equations

In UKF, nonlinear equation of process model and measurement model are as follow:

$$x_k = f(x_{k-1}, u_{k-1}) + w_k \quad (9)$$

$$z_k = h(x_k) + v_k \quad (10)$$

where x_k , u_k and z_k are state vector, input vector and measurement vector, respectively. Process noise, w_k, and measurement noise, v_k, are assumed to be random process with zero-mean Gaussian white noise with covariance matrices Q_k and R_k, respectively.

3.2 UKF Algorithm

In UKF, the predicted and measurements states, and the associated covariance matrices are computed by using sigma points and the nonlinear equations. The superscript '-'means predicted value. The UKF step can be calculated as presented in [18,19] as follows;

I. Initialization step defined

$$x_0^- = E[x_0] = \begin{bmatrix} \boldsymbol{x}_0 & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}^T \quad (11)$$

$$P_0^- = E[(x_0 - x_0^-)(x_0 - x_0^-)^T]$$
$$= \begin{bmatrix} E\left[\left(\boldsymbol{x}_0 - \hat{\boldsymbol{x}}_0\right)\left(\boldsymbol{x}_0 - \hat{\boldsymbol{x}}_0\right)^T\right] & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Q}_0 & \boldsymbol{0} \\ \boldsymbol{0} & 0 & \boldsymbol{R}_0 \end{bmatrix} \quad (12)$$

II. Time update step computed

a. 2n sigma points at *k-1* time step

$$x_{k-1}^{(i)} = x_{k-1}^+ + \tilde{x}^{(i)}, \; i = 1,2,\ldots,2n \quad (13)$$

where $\tilde{x}^{(i)} = \left(\sqrt{nP_{k-1}^+}\right)_i^T, \; i = 1,2,\ldots,n \quad (14)$

$$\tilde{x}^{(n+i)} = -\left(\sqrt{nP_{k-1}^+}\right)_i^T, \; i = 1,2,\ldots,n \quad (15)$$

n are quantity state of vector x_k .

b. 2n sigma points at *k* time step

$$x_k^{(i)} = f(x_{k-1}^i, t_k) \quad (16)$$

c. Predicted state vector

$$x_k^- = \frac{1}{2n}\sum_{i=1}^{2n} x_k^{(i)} \quad (17)$$

d. Predicted estimate covariance matrix

$$P_k^- = \frac{1}{2n}\sum_{i=1}^{2n}\left(x_k^{(i)} - x_k^-\right)\left(x_k^{(i)} - x_k^-\right)^T + Q_{k-1} \quad (18)$$

III. Measurements update step computed

e. 2n sigma points at *k* time step

$$z_k^{(i)} = h\left(x_k^{(i)}, t_k\right) \quad (19)$$

f. Predicted measurement vector

$$z_k^- = \frac{1}{2n}\sum_{i=1}^{2n} z_k^{(i)} \quad (20)$$

g. Predicted error covariance matrix

$$P_z = \frac{1}{2n}\sum_{i=1}^{2n}\left(z_k^{(i)} - z_k^-\right)\left(z_k^{(i)} - z_k^-\right)^T + R_k \quad (21)$$

h. Predicted error cross covariance matrix

$$P_{xz} = \frac{1}{2n}\sum_{i=1}^{2n}\left(x_k^{(i)} - x_k^-\right)\left(z_k^{(i)} - z_k^-\right)^T \quad (22)$$

i. Kalman gain

$$K_k = P_{xz}P_z^{-1} \quad (23)$$

j. State vector estimation

$$x_k^+ = x_k^- + K_k(z_k - z_k^-) \quad (24)$$

k. Error covariance matrix estimation

$$P_k^+ = P_k^- - K_k P_z K_k^T \quad (25)$$

The recursive process then repeats steps *a* to *k*

3.3 UKF for Accelerometer Tilt Sensing

3.3.1 Process Model

For (7) and (8), the estimated states are roll and pitch angles. Hence define state vector as follows.

$$\boldsymbol{x} = [\theta \quad \phi]^T = [x_1 \quad x_2]^T \quad (26)$$

where $x_1 = \theta$ and $x_2 = \phi$ are the roll and pitch angle in radians, respectively.

If the sensor is stable (the tilt angles do not vary during the measurement duration), the roll and pitch angles are possible to assume that are constant. Hence, the process model are as follow:

$$\dot{\boldsymbol{x}} = 0 \tag{27}$$

where $\dot{\boldsymbol{x}} = [\dot{x}_1 \quad \dot{x}_2]^T = [\dot{\theta} \quad \dot{\phi}]^T$ and $\mathbf{F} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (28)

Although, estimating random noise are constants ($\boldsymbol{w_k} = \boldsymbol{0}$), it is a common practice to include non-zero driving noise in the model. The random noise covariance matrix $\boldsymbol{Q}_k$ can have very small values. From (27) and (28), final continuous-time process model for accelerometer tilt sensing system is

$$\begin{bmatrix} \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \phi \end{bmatrix} + \begin{bmatrix} w_x \\ w_y \end{bmatrix}_k \tag{29}$$

where $\boldsymbol{w}$ is the random noise vector where the derivatives of states equal driving noises is known as random walk processes.

The final discrete-time model is as follows.

$$\begin{bmatrix} \theta \\ \phi \end{bmatrix}_{k+1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \theta \\ \phi \end{bmatrix}_k + \begin{bmatrix} w_x \\ w_y \end{bmatrix}_k \tag{30}$$

or

$$\boldsymbol{x}_{k+1} == \boldsymbol{\Phi}_k \boldsymbol{x}_k + \boldsymbol{w}_k \tag{31}$$

where $\boldsymbol{w}$ is the random noise vector with the covariance matrix, $\mathbf{Q}_k = \begin{bmatrix} Q_x & 0 \\ 0 & Q_y \end{bmatrix}$ where Q_x and Q_y may have very small values ($\approx 10^{-7}$).

3.3.2 Measurement Model

Considering accelerometer tilt sensing system model, from (7) and (8), repeated here for convenience, is nonlinear because of the trigonometric terms.

$$\tilde{A}_x = -\sin\phi + \tilde{v}_x \tag{32}$$

$$\tilde{A}_y = \sin\theta\cos\phi + \tilde{w}_y \tag{33}$$

Eqn.(32) and (33) can be used in UKF directly. So, measurement model of UKF is as follow;

$$\boldsymbol{z}_k = \begin{bmatrix} \tilde{A}_x \\ \tilde{A}_y \end{bmatrix}_k = \begin{bmatrix} -\sin\phi \\ \sin\theta\cos\phi \end{bmatrix}_k + \begin{bmatrix} v_x \\ v_y \end{bmatrix}_k \tag{34}$$

where $\boldsymbol{v}_k$ is measurement noise vector and the associated covariance matrix $\boldsymbol{R}_k = \begin{bmatrix} R_x & 0 \\ 0 & R_y \end{bmatrix}$ can be estimated from the noise power spectral density that is the root-mean-squared (rms) noise. According to [22], for an accelerometer can be approximated as follows.

$$\text{Accelerometer rms noise} \approx \text{Noise Density} \times \sqrt{\text{Bandwidth} \times 1.6} \tag{35}$$

where the noise desity is provided in the datasheet. For ADXL335 module , the noise density of accelerometer is between $150-300\,\mu g/\sqrt{Hz}$ [10] (selected full-scale ±2g, the Analog Devices noise density parameter is $300\,\mu g/\sqrt{Hz}$) and circuit of sensor is designed to have 20Hz bandwith. So, the rms noise can be computed from:

$$\text{Accelerometer rms noise (g)} \approx 300\times10^{-6} \times \sqrt{20\times1.6} \approx 0.0017g \approx 2mg \tag{36}$$

While the accelerometer measurement noise is assumed to be Gaussian white noise, the covariance matrix $\mathbf{R}_k$ is

$$\mathbf{R}_k = \begin{bmatrix} R_x & 0 \\ 0 & R_y \end{bmatrix} = \begin{bmatrix} 0.002^2 & 0 \\ 0 & 0.002^2 \end{bmatrix} \tag{37}$$

That means if taking accelerometer measurements, its white noise variance should be approximate 2 mg.

4. Experimental

Composed of low-cost MEMS accelerometer described in section 2 and Unscented Kalman filter algorithm described in section 3, testing platform for tilt angle estimation was implemented as shown in Figure 4.1. The accelerometer that used in the paper was the analog 3-axis Accelerometer, ADXL05 chip [22], which had full scale-range ±2g. Inter-integrated-circuit (I2C) protocol was used to connect between sensor and microcontroller. ARM7 microcontroller was used as STM32F4Discovery for this system and the sampling rate was 100Hz. The sensor was mounted on the leveled table which was modulated pitched or rolled angle by DC-motor. The encoder was collected the angle that was pitched or rolled angle. Firmware of microcontroller for programing the UKF algorithm was based on Matlab/SIMULINK with Waijung Blockset. The Hardware-in-the-loop (HIL) techniques was used to recorded the data form accelerometer and encoder to PC-Computer. The actual roll/pitch angle were measured by using digital level.

5. Results and Discussion

In our work, two experiments have been done. In first experiment, testing platform is stationary. Tilt angles results are shown in Fig. 5.1 and 5.2. In this figure, tilt angle estimator with UKF has good performance in tilt measurement when the tilt angles are constant.

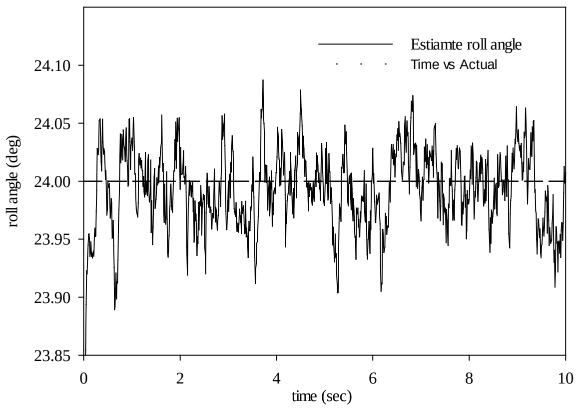

Fig 5.1 Example of Roll angle Results of 24 degree angle

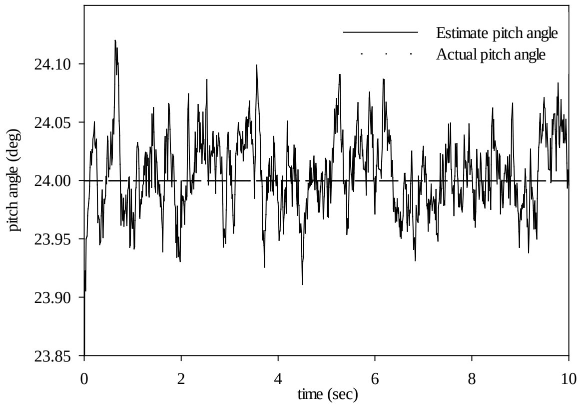

Fig 5.2 Example of Pitch angle Results of 24 degree angle

Next, performances of the UKF was Analyzed. Comparing the computed error covariance matrix $\boldsymbol{P}$ with actual errors is the simplest method to analyze performance. If 63% of the actual errors are place inside the 1 error standard deviation (S.D) bound that is squared-root values of the diagonal terms of $\boldsymbol{P}^{+}$, the UKF is suitable for estimate system states [19-21]. Performances analyzed of our UKF is shown in Fig. 5.3. More than 85% of the actual errors are lie inside the 1 error standard deviation (S.D) bound. So, our UKF for estimation of tilt angle is good performance.

In the other experiment, the testing platform varies angle on level plane and then kept constant angle. The results as show in Fig. 5.4 and 5.5. In both roll and pitch angle results, we found that UKF can be estimate the accuracy tilt angle and errors are much small. But, during the constant angle phase, error of tilt angle that error is approximately 0.2 degree is much smaller than during the transition angle phase that error is approximately 1.2 degree.

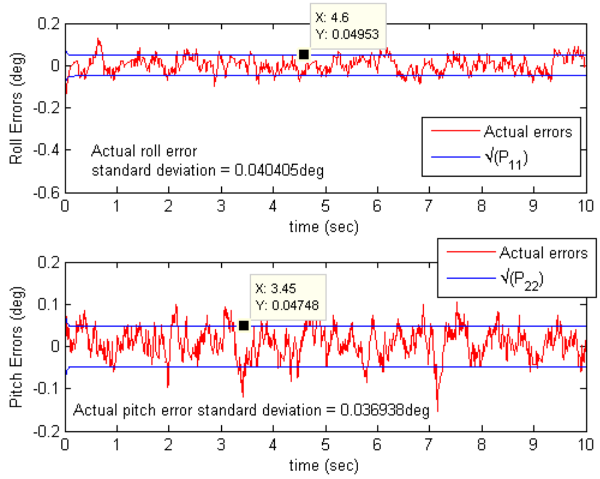

Fig. 5.3 Analyze of performances of the UKF result

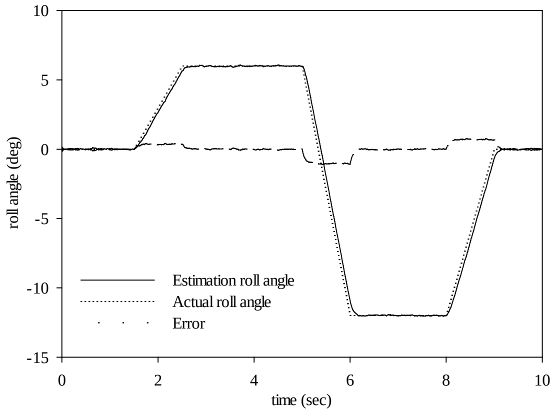

Fig 5.4 Roll Angle Testing Results. The testing platform varies angle on level plane and then kept constant angle.

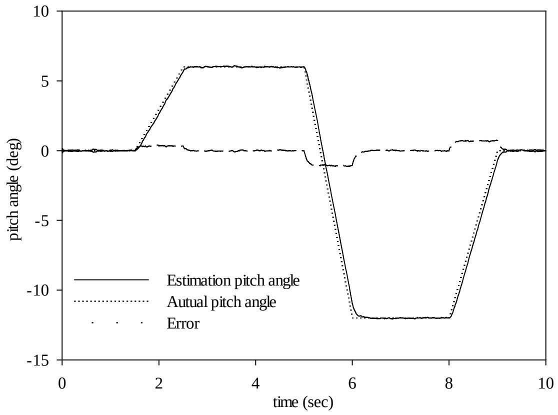

Fig 5.5 Pitch Angle Testing Results. The testing platform varies angle on level plane and then kept constant angle.

6. Conclusion

In this paper, the unscented Kalman filter implementation on MEMS accelerometer tilt sensing for estimation tilt angle was presented. The algorithm of UKF and their parameter are describes. In order to evaluate the estimation of the UKF, we used testing platform with the digital level as a reference. In our work, Experimental results have shown that the proposed method have good performance while estimating tilt angle.

7. References

[1] L. Ojeda and J. Borenstein, “FLEXnav: Fuzzy logic expert rule-based position estimation for mobile robots on rugged terrain,” in *Proc. IEEE Int. Conf. Robot. Autom.*, Washington, DC, May 2002, vol. 1, pp. 317–322.

[2] J. Hollingum, “Robots explore underground pipes,” *Ind. Robot*, vol. 25, no. 5, pp. 321–325, 1998.

[3] F. Gustafsson, M. Drevoe, and U. Forssell, “Methods for estimating the roll angle and pitch angle of a two-wheeled vehicle, system and a computer program to perform the methods,” Eur. Patent WO0201151A1, Jan. 3, 2002.

[4] M. N. Norgia, I. Boniolo, M. Tanelli, S. Savaresi, and C. Svelto, “Optical sensors for real-time measurement of motorcycle tilt angle,” *IEEE Trans. Instrum. Meas.*, vol. 58, pp. 1640–1649, May 2009.

[5] M. Wang, Y. Yang, R. R. Hatch, and Y. Zhang, “Adaptive filter for a miniature MEMS based attitude and heading reference system,” in *Proc. Position Location Navig. Symp.*, 2004, pp. 193–200.

[6] J. Vˇcelák, P. Ripka, J. Kubík, A. Platil, and P. Kaˇspar, “AMR navigation systems and methods of their calibration,” *Sens. Actuators A, Phys.*, vol. 123–124, pp. 122–128, Sep. 2005.

[7] H. J. Luinge and P. H. Veltink, “Inclination measurement of human movement using a 3-D accelerometer with autocalibration,” *IEEE Trans. Neural Syst. Rehab. Eng.*, vol. 12, pp. 112–121, Mar. 2004.

[8] A. Godfrey, R. Conway, D. Meagher, and G. ÓLaighin, “Direct measurement of human movement by accelerometry,” *Med. Eng. Phys.*, vol. 30, no. 10, pp. 1364–1386, 2008.

[9] J. Cervik, H. H. Fields, and G. Aul, “Rotary drilling holes in coalbeds for degasification,” Pittsburgh Mining and Safety Research Center, Pittsburgh, PA, Tech. Rep., 1975.

[10] J. E. Mercer, P. H. Hambling, R. Zeller, S. S. Ng, G. W. Brune, and L. A. Moore, “System for tracking and/or guiding an underground boring tool,” U.S. Patent US006035951A, Mar. 14, 2000.

[11] Horton M., Kitchin C.: A dual axis tilt sensor based on micromachined accelerometers, Sensors (1996) vol. 13, no. 4, pp. 91–94

[12] Sensors & Sensory Systems Catalog, Crossbow, San Jose, CA 2006, pp. 61–71

[13] Luczak S., Oleksiuk W., Bodnicki M.: Sensing tilt with MEMS accelerometers, IEEE Sensors J. (2006) vol. 6, no. 6, pp. 1669–1675

[14] Siouris, G. M., “Aerospace Avionics Systems: A Modern Synthesis”, 1st ed., Academic Press, Inc., 1993, pp. 20-22.

[15] STMicroelectronics, LIS3L06AL MEMS INERTIAL SENSOR: 3-axis - +/-2g/6g ultracompact linear accelerometer datasheet, May 2006. pp.5.

[16] Kalman, R.E. 1960. A new approach to linear filtering and prediction problems, Transaction of the ASME, Journal of Basic Engineering: 35-45

[17] Leonard, A. McGee and Stanley F. Schmidt. November 1985. Discovery of the Kalman Filter as a practical tool for aerospace and industry, NASA Technical Memorandum 86847

[18] Sorenson, Harold. W. 1985. Kalman Filtering: Theory and Applications. IEEE Press

[19] Brown, R. G. and Hwang, Y. C., “Introduction to random signals and applied Kalman filtering”, John Wiley & Sons, 1997, pp.214-20.

[20] Julier, S. J. and Uhlmann, J. K., Unscented Filtering and Nonlinear Estimation, Proceedings of The IEEE, Vol. 92, No. 3, March 2004., 9. Simon,

[21] D., “Optimal State Estimation”, Wiley Interscience, 2006., pp. 448-51.

[22] Analog Devices, Accelerometer ADXL335 Datasheet, Rev.0., 2009.