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Synergetic Control for Double Inverted Pendulum on a Cart

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Abstract

The double inverted pendulum on a cart is one of the classical unstable underactuated systems. In order to stabilize the system, various control techniques have been proposed. One of the applicable control methods is the sliding mode control. Even though the method can stabilize the system, the control system is affected by the chattering phenomena. The purpose of this study; therefore, was to employ the synergetic controller to the double inverted pendulum system. Furthermore, the set of controller parameters was determined systematically by the ant colony optimization (ACO). To validate the effectiveness of the synergetic controller, the simulation of the double inverted pendulum system under the synergetic controller was carried out, and the results were compared with those of the sliding mode controller. The simulation results showed that the synergetic control could stabilize the system, and the chattering phenomena in control input signal could be reduced. In conclusions, the synergetic controller with ACO can be employed successfully on the double inverted pendulum on a cart.

Keywords: Double inverted pendulum, Feedback control, Synergetic control, Ant colony optimization.

1. Introduction

The double inverted pendulum on a cart is one of the underactuated systems which, has been interested by many researchers for a number of years, [1-7]. Stabilization of the double inverted pendulum system is the challenge problem in the control area as several developed control methods have been employed to stabilize this system [1-7]. The synergetic control method is one of the interesting control methods applicable for various dynamical systems. Previous works, related the synergetic control method in both theoretical aspects and applications, have been presented as seen in literature [8-20]. The development of the synergetic control theory initially was proposed by Kolesnikov and colleges [10, 16, 17]. The main favorable aspect of the method is the smooth control signal [9, 11, 15]. This characteristic is an advantage over the sliding mode control (SMC) which has the chattering in the control signal [5-7, 9, 11, 15, 21]. In addition, the synergetic controlled system with appropriate macro variables can have the following desirable characteristics: 1) global stability, 2) parameter insensitivity, and 3) noise suppression [8]. Kolesnikov [12] applied this technique to stabilize the single inverted pendulum on a cart. In general, the selection or tuning of the optimal control parameters can be achieved by using the optimization algorithms such as the particle swarm optimization (PSO) and the genetic algorithm (GA), [5,19-20,22]. One of the well-known optimization algorithms is the ant colony optimization (ACO). The development and use of the ACO have been presented in previous works [23-29]. The relevant strength of the ACO method is from the distributed computation [27-28]. To the best of authors' knowledge, applying the synergetic control to stabilize the double inverted pendulum has not been proposed. Thus, an investigation of the ability of the SC method with ACO to stabilize the double inverted pendulum system is the main focus of this study. The simulation of the control system was used to present the ability of the SC method for the pendulum system.

The organization of this paper is presented as follows. The mathematical model of the double inverted pendulum on a cart is first presented in Section 2. The details of controller design are provided in Section 3. Then, the simulation results of the study are presented and discussed in Section 4. Finally, the conclusion from this study is stated in Section 5.

2. Mathematical Model

The mathematical model of the double inverted pendulum on a cart is derived and presented in [2]. This model of the system and related information from [2] is used in this study. The equation of motion and the state space representation of the system are presented in the following two subsections.

2.1 Equation of motion

The equation of motion of the double inverted pendulum on a cart in Fig. 1 can be presented in terms of the displacement of the cart (θ_0), angular displacement of the lower pendulum (θ_1), and angular displacement of the upper pendulum (θ_2) as Eq.(1), [2]:

$$\underline{D}(\theta)\ddot{\underline{\theta}}+\underline{C}(\theta,\dot{\theta})\dot{\underline{\theta}}+\underline{G}(\theta)=\underline{H}u\,, \qquad (1)$$

where

$$\underline{D}(\theta)=\begin{bmatrix} d_1 & d_2\cos(\theta_1) & d_3\cos(\theta_2) \\ d_2\cos(\theta_1) & d_4 & d_5\cos(\theta_1-\theta_2) \\ d_3\cos(\theta_2) & d_5\cos(\theta_1-\theta_2) & d_6 \end{bmatrix}$$

$$\underline{C}(\theta,\dot{\theta})=\begin{bmatrix} 0 & -d_2\sin(\theta_1)\dot{\theta}_1 & -d_3\sin(\theta_2)\dot{\theta}_2 \\ 0 & 0 & d_5\sin(\theta_1-\theta_2)\dot{\theta}_2 \\ 0 & -d_5\sin(\theta_1-\theta_2)\dot{\theta}_1 & 0 \end{bmatrix}$$

$$\underline{G}(\theta)=\begin{bmatrix} 0 \\ -f_1\sin(\theta_1) \\ -f_2\sin(\theta_2) \end{bmatrix},\ \underline{H}=\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\ \text{and}\ \underline{\theta}=\begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \end{bmatrix}.$$

The terms $d_1, d_2 ...., d_6$, f_1, and f_2 are defined as follows [2]: $d_1=m_0+m_1+m_2$, $d_2=(\frac{1}{2}m_1+m_2)L_1$, $d_3=\frac{1}{2}m_2L_2$, $d_4=(\frac{1}{3}m_1+m_2)L_1^2$, $d_5=\frac{1}{2}m_2L_1L_2$, $d_6=\frac{1}{3}m_2L_2^2$, $f_1=(\frac{1}{2}m_1+m_2)L_1g$, and $f_2=\frac{1}{2}m_2L_2g$, where m_0, m_1, and m_2 represents the masses of the cart, the lower pendulum, and the upper pendulum respectively. The length of the lower and the upper pendulum are denoted by L_1 and L_2 respectively.

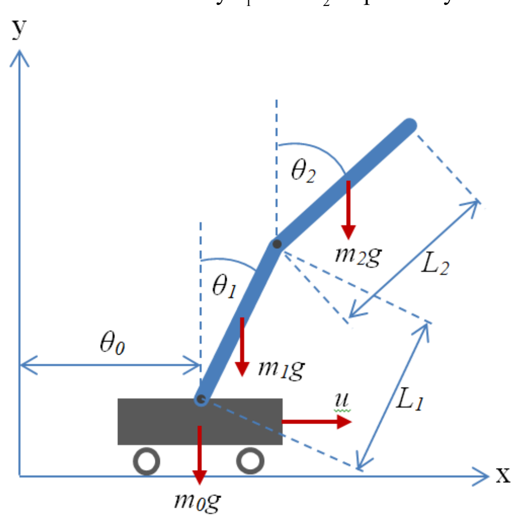


Fig. 1 Double inverted pendulum on a cart, [2].

2.2 State space representation

Based on Eq. (1), the state space representation of the double inverted pendulum can be presented in term of the state vector, $\underline{x}=\begin{bmatrix}\theta \\ \dot{\theta}\end{bmatrix}$, as Eq.(2), [2]:

$$\dot{\underline{x}}=\begin{bmatrix} \underline{0}_{3\times3} & \underline{I}_{3\times3} \\ \underline{0}_{3\times3} & -\underline{D}^{-1}\underline{C} \end{bmatrix}\underline{x}+\begin{bmatrix} \underline{0}_{3\times1} \\ -\underline{D}^{-1}\underline{G} \end{bmatrix}+\begin{bmatrix} \underline{0}_{3\times1} \\ -\underline{D}^{-1}\underline{H} \end{bmatrix}u\,. \quad (2)$$

The linearized state space model can be obtained from linearizing Eq. (2) around the equilibrium point. Thus, the linearized state space representation can be expressed as Eq. (3), [2]:

$$\dot{\underline{x}}=\underline{A}\underline{x}+\underline{B}u\,, \qquad (3)$$

where $\underline{A}=\begin{bmatrix} \underline{0}_{3\times3} & \underline{I}_{3\times3} \\ -\underline{D}(0)^{-1}\dfrac{\partial \underline{G}(0)}{\partial(\theta)} & \underline{0}_{3\times3} \end{bmatrix}$, $\underline{B}=\begin{bmatrix} \underline{0}_{3\times1} \\ \underline{D}(0)^{1}\underline{H} \end{bmatrix}$

and $\underline{x}=\left[x_1\ x_2\ x_3\ x_4\ x_5\ x_6\right]^T=\left[\theta_0\ \theta_1\ \theta_2\ \dot{\theta}_0\ \dot{\theta}_1\ \dot{\theta}_2\right]^T$.

Readers can find more details about the mathematical model of the system in [2].

3. Controller Design

The synergetic controller design for the double inverted pendulum on a cart is presented in this section, together with the ant colony optimization algorithm for control parameter determination.

3.1 Design of synergetic control

Based on [8-12, 15], the design procedure of the synergetic control for the double inverted pendulum on a cart can be presented as follows.

In the first step of the design procedure, the macro variable, ψ, is defined as Eq. (4), [8-9, 11, 15]:

$$\psi=s(\underline{x})=\underline{C}_H\,\underline{x} \qquad (4)$$

where $\underline{C}_H=[C_1\ \ C_2\ \ C_3\ \ C_4\ \ C_5\ \ C_6]$. The values of C_1, C_2, C_3, C_4, C_5, and C_6 are real constants. Consequently, the derivative of the macro variable in (4) can be presented as Eq. (5), [8-9, 11, 15]:

$$\dot{\psi}=s_x(\underline{x})\dot{\underline{x}}=\underline{C}_H\dot{x}\,, \qquad (5)$$

where $s_x(\underline{x})=\left(\dfrac{\partial s(\underline{x})}{\partial \underline{x}}\right)^T$, [11, 15].

In the second step, the dynamic evolution is defined to confine the state variables of the system as Eq. (6), [8-9, 11, 15]:

$$T\dot{\psi}+\psi=0\,, \qquad (6)$$

where T is the positive value. The selection of T is from the designer [8-9, 11, 15]. Under the control input, the convergent rate of the state variables to $\psi=0$ depends on the value of T [8-9, 11, 15].

In the last step, determination of control, u, is performed. Substituting Eq. (3) - (5) into Eq. (6), the control input can be solved as Eq. (7), [8-9, 11, 15]:

$$u=-(\underline{C}_H\underline{B})^{-1}\underline{C}_H\underline{A}x-(\underline{C}_H\underline{B})^{-1}T^{-1}s \qquad (7)$$

According to [11, 15], the proof of the closed-loop system stability under the SC method can be presented as follows. First, the Lyapunov function is written in term of the macro variable, ψ, as Eq. (8), [11, 15]:

$$V = 0.5\psi^2 \tag{8}$$

The derivative of the Lyapunov function can be determined as Eq. (9), [11, 15]:

$$\dot{V} = \psi\dot{\psi} \tag{9}$$

Based on Eq. (4) and (5), $\dot{V}$ can be written as Eq. (10), [11, 15]:

$$\begin{aligned} \dot{V} &= s(\underline{x})(s_x(\underline{x})\dot{\underline{x}}) \\ &= s(\underline{x})s_x(\underline{x})[\underline{A}\underline{x} + \underline{B}u] \end{aligned} \tag{10}$$

Then, evaluating control input, u, in Eq. (7) into Eq. (10), the derivative of the Lyapunov function, $\dot{V}$, can be expressed as Eq. (11), [11, 15]:

$$\begin{aligned} \dot{V} &= s\underline{C}_H[\underline{A}\underline{x} + \underline{B}(-(\underline{C}_H\underline{B})^{-1}\underline{C}_H\underline{A}\underline{x} \\ &\quad -(\underline{C}_H\underline{B})^{-1}T^{-1}s)] \\ &= -sT^{-1}s \\ &= -T^{-1}s^2 \le 0\,. \end{aligned} \tag{11}$$

Equation (11) implies that the control system is stable [11, 15]. Thus, the control input in Eq. (7) can stabilize the double inverted pendulum on a cart. The state variables of the control system can be driven to the equilibrium point. More details on the method can be further determined from [8-12, 15-17].

It is important to note that the synergetic control (SC) method, and the sliding mode control (SMC) method have the same important characteristic in terms of the equivalent control; therefore, the surface function $s(\underline{x})$ or sliding surface defined for the SMC method can be used for the synergetic control (SC) [5, 9, 11, 15]. Consequently, the techniques such as LQR and pole placement used in the SMC method can be utilized to determine the coefficients, $C_1,...,C_6$, for the macro variable, $\psi = s(\underline{x})$, in Eq.(4), [5, 7-9, 11, 15, 21, 30-31].

3.2 Ant colony optimization (ACO)

Some optimization algorithms have been employed with the synergetic control methods for tuning the controller parameters [19-20]. For example, the particle swarm optimization was used with the synergetic control (SC) in previous works [19-20].

The Ant colony optimization (ACO) algorithm was developed by Dorigo and colleagues and was inspired by ant activities to search the food with the optimal paths and presented in previous works [23-29]. In the ACO algorithm, artificial ants are used to construct solutions by exploring on a set of optimization variables [27-29]. The movement of ants depends on the pheromone given by other ants [27-29]. The ACO is a technique to find the approximated solutions of various optimization problems [27-28]. The ACO was used to find the optimal gain of the state feedback controller for the single inverted pendulum [29].

In order to determine the controller parameters by using ACO, these parameters are considered as the optimization or decision variables [5, 19, 29].

In this study, the ACO was applied to tune the controller parameters of the SC method including $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6$, and T [5, 19, 29]. The optimal values of $C_1,...,C_6$ can be found from optimal eigenvalues [5, 21, 31]. Based on the tuning the parameters of the controller by using optimization algorithm and the ant colony optimization [5, 19, 27-29], the steps of ACO algorithm using in this paper are presented as follows: Step 1: The ACO parameters and the pheromone matrix are initialized. Step 2: The searching space nodes of each optimization variable are constructed. Step 3: The random allocation of ants to the variable nodes is performed. Step4: The simulation of the controlled double inverted pendulum system with the corresponding controller parameters from each ant is carried out. Step 5: The cost function in Eq. (12) is computed. Also, if the cost function is minimum, the set of the best controller parameters is updated. Step 6: The change of the pheromone is computed and the pheromone matrix is updated. Step 7: Steps 4 to 6 are repeated for each ant. Step 8: Step 3 to 7 are repeated for N iterations.

The ACO is performed to search for the optimal value of the set of controller parameters to minimize the cost function which is the integral time absolute error (ITAE) as Eq. (12), [19, 22, 27-29]:

$$J = \int_0^t t\left|e(t)\right|dt \tag{12}$$

4. Simulations

The example of the double inverted pendulum system under the synergetic control is presented in subsection 4.1. Then, the simulation results and discussion are presented in subsection 4.2.

4.1 Simulation example

The parameters of the double inverted pendulum on a cart which are masses and length of the pendulums and the cart in Eq.(3) can be presented as follows [2]: i) $m_0 = 1.5\ kg$, $m_1 = 0.5\ kg$, and $m_2 = 0.75\ kg$, ii) $L_1 = 0.5\ m$ and $L_2 = 0.75\ m$. The initial condition of the system was assumed as $\underline{x}(0) = [0.0100\ 0.1047\ 0.1047\ 0\ 0\ 0]^T$. The disturbance was the Gaussian function, $d(t) = a_d e^{-(t-c_d)^2/(2b_d^2)}$, where $a_d = 0.5$, $b_d = 0.5$, and $c_d = 10$.

The control parameters $C_1,...,C_6$ could be determined by using the linear quadratic regulator

(LQR) algorithm [21, 30]. The matrices of Q and R for the LQR algorithm were defined as Eq. (13), [21, 30]:

$$\underline{Q} = \begin{bmatrix} 400 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 500 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } \underline{R} = 1. \quad (13)$$

The numeric values of C_1, C_2, C_3, C_4, C_5, and C_6 given by LQR optimization are 20.0000, -387.5010, 628.820, 39.9246, 7.6765, and 101.1541 respectively. The ACO algorithm was performed for 10 iterations with 100 ants over the interval of T from 0.05 to 0.1. The value of T yielded by the ACO is 0.078258.

The synergetic control (SC) method was applied to the pendulum system. Then, the simulation results corresponding to the SC method were compared to those of the sliding mode control (SMC) method. The SMC method used in this study is presented as Eq. (14), [11, 21, 31]:

$$u = -(\underline{C}_H \underline{B})^{-1} \underline{C}_H \underline{A}x - (\underline{C}_H \underline{B})^{-1} \eta sign(s), \quad (14)$$

where η is the design parameter [11,21,31]. The value of η was selected as $\eta = 10$.

In order to demonstrate the ability of the ACO for the controller parameter determination, the ACO was employed to determine the controller parameters which are $C_1,...,C_6$, and T. Since the values of $C_1,...,C_6$ could be found from the eigenvalues, $\lambda_1,....,\lambda_6$; therefore, these eigenvalues were considered as the optimization variables for ACO [5,21,31]. The range of each eigenvalue was from -10 to -4, and T was constrained within the range from 0.05 to 0.1. After performing the ACO with 100 ants for 10 iterations, the optimal values of controller parameters were determined as follows: i) T=0.073407 ii) λ_1=-9.2697, λ_2=-10.0000, λ_3= -6.6781, λ_4=-4.6467, λ_5=-4.8455, and λ_6= -9.5860. Thus, the corresponding values of the parameters of the controller, $C_1,...,C_6$, could be determined as C_1=0.2025 $\times 10^3$, C_2=-0.3379 $\times 10^3$, C_3=1.1373 $\times 10^3$, C_4=0.1789 $\times 10^3$, C_5=0.0821 $\times 10^3$ and C_6=0.2083 $\times 10^3$ by using Ackermann's formulas [5, 21, 31].

In simulation, the Runge-Kutta method was used for the numerical integration and performed from $t = 0$ to $t = 70$ seconds with the time step of 0.01 seconds.

4.2 Simulation results and discussion

The simulation results of the double inverted pendulum on a cart system under the synergetic control with controller parameter T given by the ACO and sliding mode control are presented in Figs. 2 and 3 respectively. The time responses of the double inverted pendulum system under the SC method are presented in Fig. 2(a), while those of the SMC method are presented in Fig. 2(b). The control input of both methods are plotted and shown in Fig. 3(a). The zoom-in plot of the control input is shown in Fig. 3(b).

Additionally, the simulation results of the double inverted pendulum system manipulated by the SC method with the optimal controller parameters of $C_1,...,C_6$, and T yielded by ACO are presented in Figs. 4 and 5. The time responses of all state variables of the control system are presented in Fig. 4. The corresponding SC control input is shown in Fig. 5.

Considering Fig. 2(a), all state variables corresponding to all of the links of the double inverted pendulum system under the SC method were driven to zero as the time increased. The synergetic control could stabilize the pendulum system. Fig. 2(b) showed that the SMC method could also stabilize the system. However, the control input of the SC method was smoother compared to that of SMC method which contains the chattering as seen in Figs. 3(a) and 3(b).

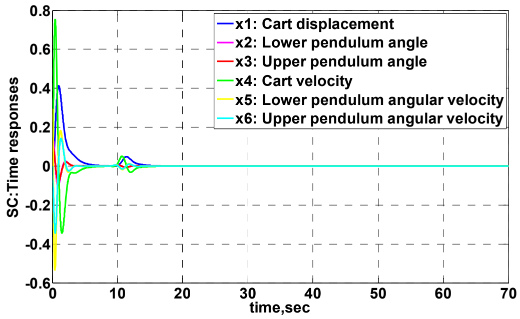
In the case when the ACO was used to determine all controller parameters of the SC method, the SC method could stabilize the double inverted pendulum system as clearly seen in Fig. 4. The preferable characteristic of the control input can be seen in Fig. 5. Thus, the ACO can provide a systematic way for designers to determine the parameters of the SC method as applying optimization algorithms for this purpose, as shown in previous works [5, 19-20, 22, 29].

The control signals in Figs. 3 and 5 showed that the SC method could stabilize the considered double inverted pendulum system with the smooth control. In practical situations, the smoothness of the control signal is a preferable characteristic, while the chattering phenomenon needs to be avoided or reduced [5-7, 9, 11, 15, 21].

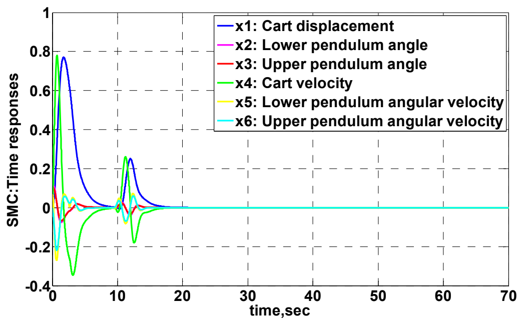
Therefore, it is clear that the SC method with the ACO controller parameter tuning can be applied for stabilizing of the double inverted pendulum on a cart system.

5. Conclusion

The conclusion can be summarized as follows. First, the synergetic controller can stabilize the double inverted pendulum on a cart. Second, the use of ACO algorithm provides the systematic way for the designer to determine the designed parameter for the SC method. Third, the SC method can manipulate the pendulum system with the smooth control and provide the improvement in terms of the chattering reduction. Thus, the synergetic control with ACO is an acceptable method to stabilize the double inverted pendulum on a cart.

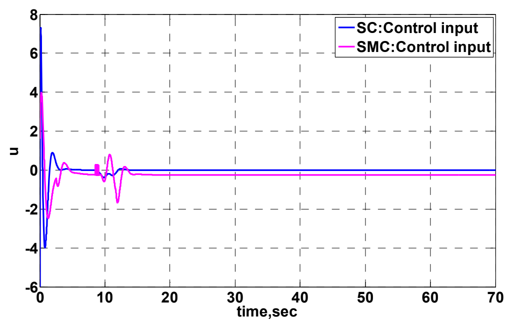


(a)

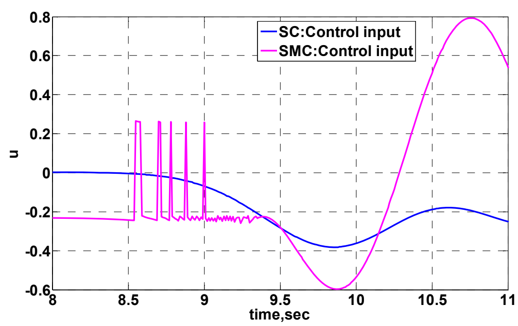


(b)

Fig. 2 Time responses of the control system: (a) the SC method with the controller parameter T given by the ACO (b) the SMC method.



(a)



(b)

Fig. 3 Control inputs of the SC and SMC methods: (a) The plot of control signals for $0 \leq t \leq 70$ sec (b) The zoom-in plot of control signals.

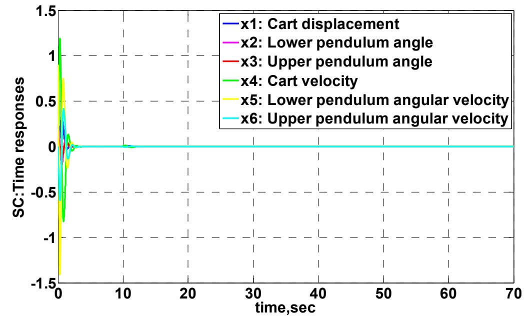


Fig. 4 Time responses of the control system under the SC method with all controller parameters given by the ACO.

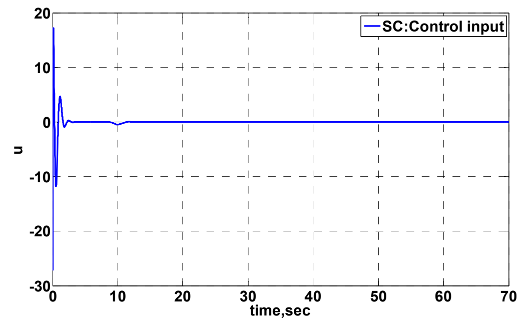


Fig. 5 The control input of the SC method with all controller parameters given by ACO.

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