Mathematical modelling of co-digestion of oil palm empty fruit bunches with palm oil mill effluent for biogas production.

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Abstract

Co-digestion of palm oil mill effluent and empty fruit bunch for biogas production gain an interest recently. However, a proper mathematical model of the digestion is essential for system design. This paper presents curve fitting of data from one of the most cited literature using Microsoft Excel 2007 and Add-in tool. The models were fitted to the experimental data of cumulative yield of methane. The coefficient of determination (R^2) and root mean square error (RMSE) were used to determine the best model. It was found that the Morgan model was the best in comparison the commonly used Modified Gompertz model.

Keywords: Mathematical modeling, Co-digestion, curve fitting

1. Introduction

After Malaysia and Indonesia, Thai palm oil industry was ranked the third in the world production. In 2012, total planting area was 4.28 million rai of which 3.98 million rai can be cultivated. Total palm oil production was about 1.9×10^6 tones [1]. From the milling process, the following biomasses are generated, empty fruit bunch (EFB), palm press fiber (PFB) and shell. The corresponding figures are 0.24 t-EFB/t-FFB, 0.18 t-PFB/t-FFB and 0.08 t shell/t-FFB. In addition, wastewater is discharged at 0.546 m^3/t-FFB, which is known for short as POME (palm oil mill effluent) [2].

Biogas is fermentation by anaerobic digestion with anaerobic bacteria of biodegradable materials such palm oil mill effluent, green waste, and crops. Biogas production had 4 steps (Figure 1). Biogas contain of methane (CH_4) 50-75%, carbon dioxide (CO_2) 25-50% and may have small amounts of hydrogen sulphide (H_2S) 0-3% [3]. Factors and parameter of the real biogas production is temperature, volatile solid, nonwood lignocellulosic material, organic matter and anaerobic digestion.

At present, Thailand has a policy that drives the energy generated from renewable sources. This provides opportunity for biogas production from POME. Many mills installed biogas production systems and sell electricity to national grid. Due to its high moisture content (64.5%), huge amount of EFB was left-over at the mills.

Selling to nearby mushroom farms might be a practical alternative, but co-digestion to biogas production is attractive in term of economic return [4].

Co-digestion of EFB and POME is a promising opportunity to increase biogas production. O-Thong, *et. al.,* (2012) reported graphical results of POME and POME-EFB co-digestion, which is not yet in a proper form for further studies, especially for production-utilization simulation to enable the mills to run the system effectively. There are no reported about co-digestion in plant of Thailand.

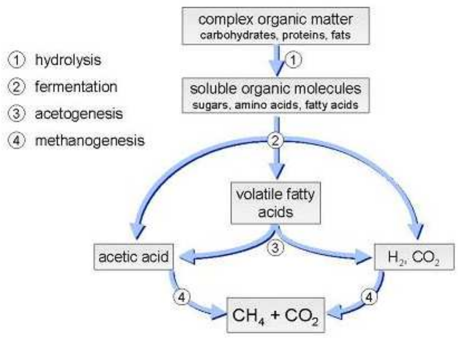

Figure 1 Biogas production [3].

In order to fully understanding the kinetics of biogas production, mathematical models, which are generally obtained empirically from experiment results, is essential. Borja et., al. (2005) demonstrated that the fifth-order polynomial expression well represented the cumulative methane production in biodegradation of two-phase olive mill solid waste. A study on grass and pig manure co-digestion yielded results that fit modified Gompertz equation (Dechrugsa and Chaiprapat, 2012). It seems that the modified Gompertz equation was chosen to represent methane production by many researchers recently when applied to different substrates for instances, apple waste and swine manure (Kafle and Kim, 2012), fat oil grease and synthetic kitchen waste (Li, et., al., 2012), pig waste and paper sludge (Parameswaran and Rittmann, 2013). This work is an attempt to mathematically rewrite the results and compare with various functions obtained from literatures.

2. Materials and method

Co-digestion of POME and EFB from experimental results reported by O-Thong (2012) (Figures 2 and 3) is used to construct mathematical models for comparison. Table 1 gives 13 equations by NCSS Statistical Software that will be tested against Figures 2 and 3.

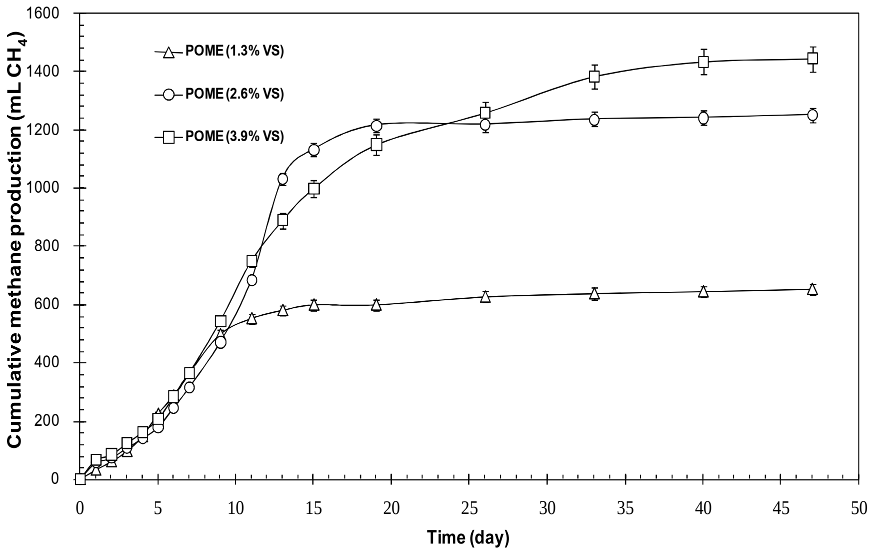

Figure 2 Cumulative methane production of POME [4]

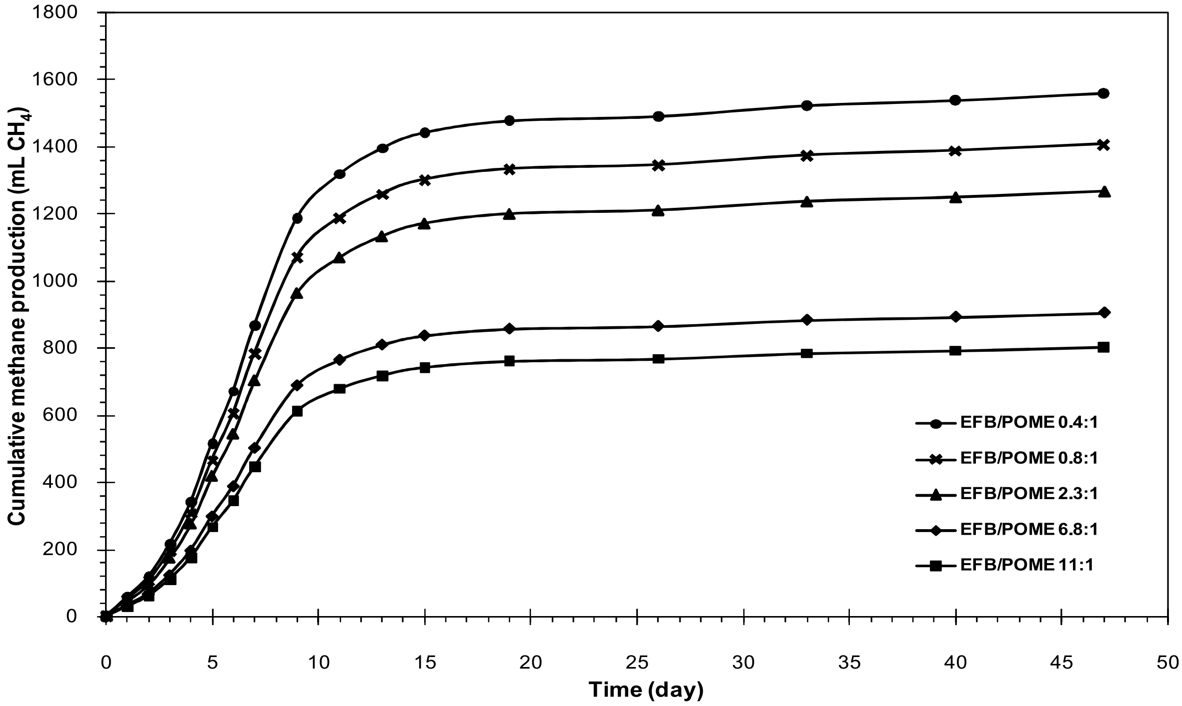

Figure 3 Cumulative methane production for co-digestion of POME and EFB [4]

Table 1 Possible mathematical models for Cumulative methane production [10].

No	model	Name of the model
1	$y = a + \left[\frac{b-a}{1+\left(\frac{x}{c}\right)^d}\right]$	Morgan-Mercer-Floding model
2	$y = a * \left[1+(b-1)\exp^{-c(x-d)}\right]^{\left(\frac{1}{1-b}\right)}$	Richards model 1
3	$y = \frac{a}{1+\exp^{-b(x-c)}}$	Logistic curve
4	$y = \frac{1}{c+ab^x}$	Logistic curve(Verhursh,1830)
5	$y = P * \exp\left\{-\exp\left(\left[\frac{R_{max} * e}{P}\right] * (\lambda - x) + 1\right)\right\}$	Modified Gompertz model (Gibbson etc.)
6	$y = \frac{k}{1+\exp^{-\alpha x-\beta}}$	Sigmoid Function
7	$y = \frac{a}{1+b*\exp^{-cx}}$	Three parameter Logistic curve Model
8	$y = d + \left[\frac{a-d}{1+b*\exp^{-cx}}\right]$	Four parameter Logistic curve Model
9	$y = a * \left[\exp\left(-\exp\left(-b(x-c)\right)\right)\right]$	Gompertz model
10	$y = \frac{a}{(1+b(\exp(-c*x)^{\left(\frac{1}{d}\right)})}$	Richards model 2
11	$y = a - (b*\exp(-c*(x^d))$	Weibull model
12	$y = A + (\frac{b*x^d}{c^d+x^d})$	Hill model
13	$y = \frac{(a*b)+(c*x^d)}{b+x^d}$	MMF model

3. Results and Discussion

Figure 2 gives cumulative methane production from POME which had volatile solid varying from 1.3% to 3.9%. The results of co-digestion for different ratio of EFB and POME are shown in Figure 3. Pattern of the results indicated 3 stages, namely the startup period, growth period and final saturation period, which is normally known as S-curve [11]. MS Excel with add-in tool was implemented to determine mathematical models using non-linear regression analysis curve-fitting technique. The curve fitting results were justified by R^2 and RMSE (root mean square error) given by Equations (1) and (2).

$$R^2 = 1 - \frac{\sum (Y_{exp} - Y_{fit})^2}{\sum Y_{exp}^2 - \frac{(\sum Y_{exp})^2}{n}} \tag{1}$$

$$RMSE = \sqrt{\frac{(Y_{exp} - Y_{fit})^2}{n}} \tag{2}$$

When, Y_{exp} is experimental cumulative methane production

Y_{fit} is predicted cumulative methane production

N is number of data points

Comparing among 13 models in Table 1 the best models representing POME and co-digestion are shown in Tables 2. It was found that Hill, Morgan and MMF agreed with most conditions while Richard 1 is better with POME at 2.6% VS. It is noted that Hill, Morgan and MMF are coincident in terms of R^2 and RMSE. The difference appeared at the seventh digit, which indicated that Morgan model is superior. Table 2 also reveals that Morgan model is more accurate that Modified Gompertz model in terms of R^2 and RMSE.

Table 2 Best-fitted models of accumulative methane production from POME and co-digestion

Condition	**Model**	**1.3% VS**		**2.6% VS**		**3.9% VS**	
		R^2	**RMSE**	**R^2**	**RMSE**	**R^2**	**RMSE**
Digestion POME	Hill	0.9981	9.9133	0.9931	40.6968	0.9988	17.7018
	Morgan	0.9981	9.9133	0.9931	40.6968	0.9988	17.7018
	MMF	0.9981	9.9133	0.9931	40.6968	0.9988	17.7018
	Richard 1	0.9973	11.7905	0.9986	18.6205	0.9972	27.0809
	Modified Gompertz	0.9967	13.0279	0.9863	57.3471	0.9971	27.4278

Condition	**Model**	**4.6% VS**									
		0.4:1		**0.8:1**		**2.3:1**		**6.8:1**		**11:1**	
		R^2	**RMSE**	**R^2**	**RMSE**	**R^2**	**RMSE**	**R^2**	**RMSE**	**R^2**	**RMSE**
CO-digestion POME with EFB	Hill	0.9989	17.6265	0.9987	18.0756	0.9989	15.1629	0.9985	12.5894	0.9987	10.3289
	Morgan	0.9989	17.6265	0.9987	18.0756	0.9989	15.1629	0.9985	12.5894	0.9987	10.3289
	MMF	0.9989	17.6265	0.9987	18.0756	0.9989	15.1629	0.9985	12.5894	0.9987	10.3289
	Modified Gompertz	0.9984	22.2344	0.9981	21.7159	0.9982	19.4628	0.9979	14.7293	0.9981	12.4090

Morgan expressions were compared with Modified Gompertz and the results were given in Table 3. The disagreement between the two models is higher during the first 5 days, which can be as high as 5.6 %. However, the difference of the final cumulative methane (at 47 days) is less than 1.5%. Finally, expressions for accumulative methane production based on Morgan model are shown in Table 4.

Table 3 Comparison of Morgan and Modified Gompertz models (difference based on Morgan model).

Substrate	Condition	Models	cumulative methane production				
			5 days	15 days	26 days	40 days	47 days
Digestion POME	1.3% VS	Morgan	220.2757	601.5232	634.3654	639.8809	640.6432
		Mod. Gompertz	228.3097	607.8402	635.0962	636.0176	636.0281
		Difference (%)	3.65	1.05	0.12	0.60	0.72
	2.6% VS	Morgan	147.0655	1090.0982	1243.3735	1255.891	1256.906
		Mod. Gompertz	151.4298	1047.9687	1249.9035	1266.0371	1266.5025
		Difference (%)	2.97	3.86	0.53	0.81	0.76
	3.9% VS	Morgan	218.0076	992.0058	1311.3972	1414.9524	1434.4336
		Mod. Gompertz	230.1518	989.0655	1333.0085	1406.9477	1412.8908
		Difference (%)	5.57	0.29	1.65	0.57	1.5
Co-digestion EFB with POME	0.4:1	Morgan	517.1158	1422.1358	1519.4989	1532.3476	1534.1126
		Mod. Gompertz	531.3606	1459.5415	1519.6577	1521.4445	1521.4621
		Difference (%)	2.75	2.63	0.01	0.71	0.82
	0.8:1	Morgan	459.4841	1302.9338	1372.8579	1384.2692	1385.8187
		Mod. Gompertz	474.7473	1317.7270	1373.5281	1375.2183	1375.2359
		Difference (%)	3.32	1.14	0.05	0.65	0.76
	2.3:1	Morgan	409.6979	1173.4142	1235.0549	1244.9653	1246.299
		Mod. Gompertz	421.9495	1187.1867	1235.0984	1236.4549	1236.4675
		Difference (%)	2.99	1.17	0.004	0.68	0.79
	6.8:1	Morgan	295.2732	836.7408	881.031	888.2054	889.1753
		Mod. Gompertz	305.2975	846.1209	881.4231	882.4771	882.4875
		Difference (%)	3.39	1.12	0.04	0.64	0.75
	11:1	Morgan	262.5628	744.5332	784.4904	791.0113	791.8967
		Mod. Gompertz	271.2841	752.9866	784.9733	785.8395	785.8492
		Difference (%)	3.32	1.14	0.06	0.65	0.76

Table 4 Best-fitted expression for methane production (Morgan model)

Substrate	Condition	Best expression
Digestion POME	1.3% VS	$CMP = 41.9023 + \left[\dfrac{599.8831}{1 + \left(\dfrac{t}{-3.1783} \right)^{6.5535}} \right]$
	2.6% VS	$CMP = 110.9519 + \left[\dfrac{1146.844}{1 + \left(\dfrac{t}{-4.7247} \right)^{10.3251}} \right]$
	3.9% VS	$CMP = 63.5928 + \left[\dfrac{1411.3593}{1 + \left(\dfrac{t}{-2.5030} \right)^{11.5568}} \right]$
Co-digestion EFB with POME	0.4:1	$CMP = 77.6812 + \left[\dfrac{1459.0574}{1 + \left(\dfrac{t}{6.5068} \right)^{-3.1954}} \right]$
	0.8:1	$CMP = 73.5119 + \left[\dfrac{1314.581}{1 + \left(\dfrac{t}{6.621} \right)^{-3.2293}} \right]$
	2.3:1	$CMP = 60.4711 + \left[\dfrac{1187.7655}{1 + \left(\dfrac{t}{6.5441} \right)^{-3.255}} \right]$
	6.8:1	$CMP = 47.5425 + \left[\dfrac{843.049}{1 + \left(\dfrac{t}{6.5528} \right)^{-3.2419}} \right]$
	11:1	$CMP = 42.0065 + \left[\dfrac{751.1898}{1 + \left(\dfrac{t}{6.5621} \right)^{-3.2292}} \right]$

4. Conclusion

To obtain accurate mathematical expression for cumulative methane production from POME and co-digestion of POME and EFB for palm oil mills, thirteen models were compared by curve fitting of experimental data. It was found that the most appropriate expression was Morgan model which the R^2 value 0.9931-0.9988 for POME digestion and 0.9985-0.9989 for co-digestion. Although the difference in comparison with the commonly used Modified Gompertz model is relatively high in the first five days of digestion (2.75 – 5.57 %), the final results (cumulative methane production) is not significantly different.

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Nomenclature

a, b, c, d, k, α and β	Parameter in cumulative methane production
e	2.71828
X or t	Day of cumulative methane production
Y or CMP	cumulative methane production
R_{max}	Maximum specific methane production rate
P	Methane production potential
λ	Lag phase time