Burgers Model Parameter Identification of Bamboo Creep Behavior

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Abstract

Bending creep behavior in the longitudinal direction of bamboo was studied. Creep and recovery tests were performed to investigate nonlinear viscoelastic strain. The long-term uniaxial creep and recovery laboratory tests were carried out at three different constant stress levels. The Burgers model was used to characterize the observed creep and creep-recovery data. The least squares method in the Levenberg-Marquardt variant was used for identifying the material model parameters. The model successfully describes the main feature for the investigated material and shows good agreement with experimental creep data.

***Keywords**:* Bamboo, Burgers Model, Creep and Recovery, Parameter Identification.

1. Introduction

Bamboo is one of the oldest construction materials due to its excellent physical and mechanical properties. For instance, in Thailand, bamboo has traditionally been used in scaffolding, low-rise houses, footbridges, and construction platform. Recently, a considerable amount of interest has been focused on bamboo because of environmental conservation and wood-based resource shortages [1]. In addition to being an eco-friendly material, bamboo is fast becoming a promising wood substitute. One of the chief reasons for this is that bamboo can be harvested in 3 to 5 years which its mechanical strength and anatomical characteristics become stable and mature [2].

Bamboo is also an anisotropic, viscoelastic, and heterogeneous material that possess high strength and stiffness along the axial direction, but shows poor mechanical properties in the transverse direction [3]. This characteristic is mainly due to the structure that composes of the lignin matrix reinforced with fibers aligned in a longitudinal direction. However, the bamboo culm is a hollow and thin-walled cylinder without fiber reinforcement in the radial direction. In addition, bamboo is a hygroscopic material that can absorb water from its surroundings. Also, bamboo is likely to be subjected to dimensional change when the ambient environment fluctuates greatly. Dimensional variation is a serious and complex phenomenon, since it can have a deleterious effect on other mechanical and physical properties [1].

Bamboo is a viscoelastic material exhibiting properties that are common to both perfect solid and perfect liquid. This properties present that stress is proportional to strain, rate of strain, and possibly higher time derivatives of strain. A constitutive equation for such material must correctly represent the behavior under conditions of stress and strain. These conditions are creep, stress relaxation, recovery, constant rate stressing and constant rate straining [4]. Because bamboo is a viscoelastic material at normal operating stress, temperature and moisture content, it is susceptible to creep which can lead to service ability problems due to excessive deformations or to safety problems due to strength reduction [5].

Various numerical models have been used to predict the viscoelastic behavior for such material. To construct the constitutive relation of the bamboo culm, a rheological effect that represents the viscoelastic behavior under various conditions of moisture content must be included. This phenomenal can be predicted by using the Burgers model as shown schematically in Fig. 1. The Burgers model is a combination of Maxwell and Kelvin-Voigt models. It is typically used for determining the relationship between the morphology of the composites and their creep behavior. For the most general case of the nonlinear viscoelastic solid, the total strain composes three essentially separate strains [6] (as shown in Fig. 2), that are the immediate elastic deformation ε_M, the delayed elastic deformation ε_{KV}, and the permanent deformation ε_P. The total strain can be written as a function of time as follow [7]:

$$\varepsilon(t) = \varepsilon_M + \varepsilon_{KV} + \varepsilon_P \tag{1}$$

$$\varepsilon(t) = \frac{\sigma}{E_M} + \frac{\sigma}{E_{KV}}\left[1 - \exp\left(-\frac{E_{KV}}{\eta_{KV}}t\right)\right] + \frac{\sigma}{\eta_M}t \tag{2}$$

where $\varepsilon(t)$ is the creep strain, σ is the applied stress, t is the time after loading, E is modulus, and η is viscosity. The subscript M refers to the properties of the Maxwell spring and dashpot, and the subscript KV is the properties of the Kelvin-Voigt spring and dashpot.

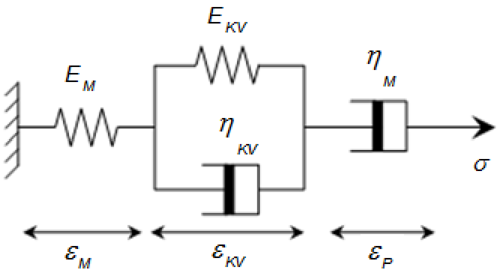

Fig. 1 Schematic of Burgers model.

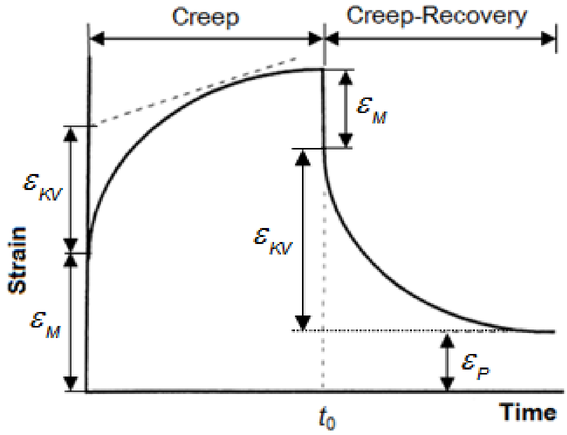

Fig. 2 Typical creep and recovery curve in a viscoelastic materials.

Xu*et al.* [8] developed the Burgers model by introducing a stretching exponent *n* to time *t* on the Kelvin-Voigt unit based on the four-element Burgers model. The model is as follow:

$$\varepsilon(t) = \frac{\sigma}{E_M} + \frac{\sigma}{E_{KV}}\left[1 - \exp\left(-\frac{E_{KV}}{\eta_{KV}}t^n\right)\right] + \frac{\sigma}{\eta_M}t \tag{3}$$

When the applied stress σ is removed at the time t = t_0, the elastic component of deformation cases instantly decrease. The creep deformation also decreases with time and tends asymptotically to the value of σ / E_M. The Weibull distribution equation [6] can be used to simulate the creep-recovery part as represented in Fig. 2. When the load is removed, some strain may be instantaneously recovered. The strain recovery can be written as:

$$\varepsilon_r(t) = \varepsilon_{KV}\left[\exp\left(-\left(\frac{t\text{-}t_0}{\eta_r}\right)^k\right)\right] + \varepsilon_P \qquad (4)$$

where the ε_{KV}, for viscoelastic strain recovery, is determined by the characteristic life (η_r) and shape (k) parameters over recovery time t, t_0 is the time which stress is removed, and ε_P is the permanent strain from viscous flow effects. Additionally, the value of ε_{KV} and ε_P can be received through the simulation of the Weibull equation.

The Burgers model of viscoelasticity has been applied for composite materials by many researchers. However, there is no publication applying the model to construct the constitutive relation for bamboo culm. For more details of using the model for other materials such as composite materials, please see paper in reference number 6, 8 and 9.

In this paper, bamboo culms are bending tested at different stress levels. Experimental results are used to construct the constitutive relation of the bamboo culm. To construct the relation, the Burgers model is used. The rheology effect is also included in the model as well.

2. Experimental

2.1 Sample preparation

The dimensions of bamboo specimen were 120 mm in length (longitudinal direction, L), 7 mm in height (radial direction, R), and 25 mm in width (transverse direction, T) as shown in Fig. 3. Approximately 3 to 4 year olds bamboo culms (*Dendrocalamus hamiltonii*) were used in this study. They were taken from Mae Wang district, Chiang Mai province. After removing the outer and the inner surface of the culm, they were polished with Silicon Carbide (SiC) sandpaper to ensure that the surface finish was in good condition. Then, the specimens were dried in an oven at 103±2^{0}C for 24 h. After 24 h, the drying was considered to be complete when the difference between the successive determinations of the mass does not exceed 0.01 g [10]. The equilibrium moisture content (EMC) of the specimens after drying was 9% EMC.

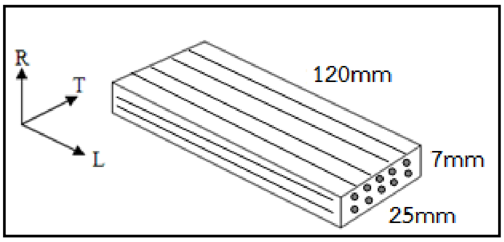

Fig. 3 Dimensional of bamboo culm specimen.

2.2 Creep characterization

The creep tests were performed on a cantilever beam system by applying a static load as shown in Figs. 4 and 5. The gauge length was 90 mm in the longitudinal direction. The experiments were conducted under 65±2% humidity and 25±2^{0}C surrounding. Tissaoui [4] suggest a creep test duration 4 hours can be sufficient to account for creep behavior of wood.

In this study, the creep tests have been tested according to the following schedule: 4 h of uniaxial bending creep at constant stress σ with three different stress levels. The three levels are σ = 20%, 30%, and 40% comparing with the ultimate strength (S_{ut}). The ultimate strength used in this study is 86.5 MPa. This strength value was stated by the authors' previous work [11]. After 4 h testing, the stress was released and the sample was allowed to recovery for 4 h. Creep-recovery strain were monitored. The experiment was repeated 3 times in each stress level. Creep deflection of the specimens was measured by using LVDT transducer with an accuracy ±0.001 mm. Data was collected with an HBM MGCplus data acquisition system.

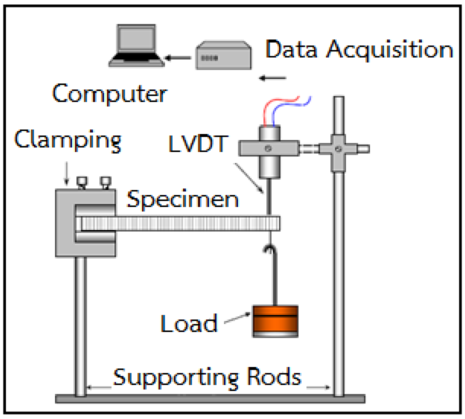

Fig. 4 Schematic of creep test rig [11].

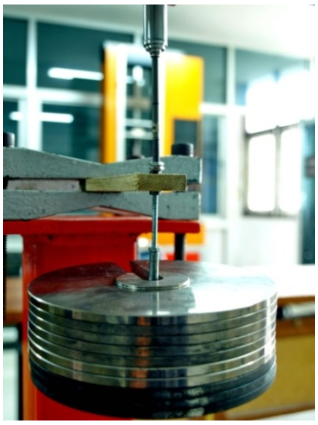

Fig. 5 Creep test photo [11].

2.3 Parameters Identification

In order to describe the viscoelastic behavior of the tested specimens, the material parameters according to the Burgers model and associating with Weibull distribution equation as shown in Eqs. (3) and (4) were constructed. This can be done by using the Levenberg-Marquardt algorithm. It is a numerical optimization algorithm for solving nonlinear equalization problems with the method of least squares. The method also combines the Gauss-Newton method with a regularization, which forces decreasing function values. The Levenberg-Marquardt algorithm is much more robust than the Gauss-Newton method, i.e., it converges with high probability even in poor starting conditions [12]. MATLAB code was employed to estimate the material parameters by performing nonlinear regression on the creep data with the Levenberg-Marquardt algorithm. The computational process is shown in Fig. 6.

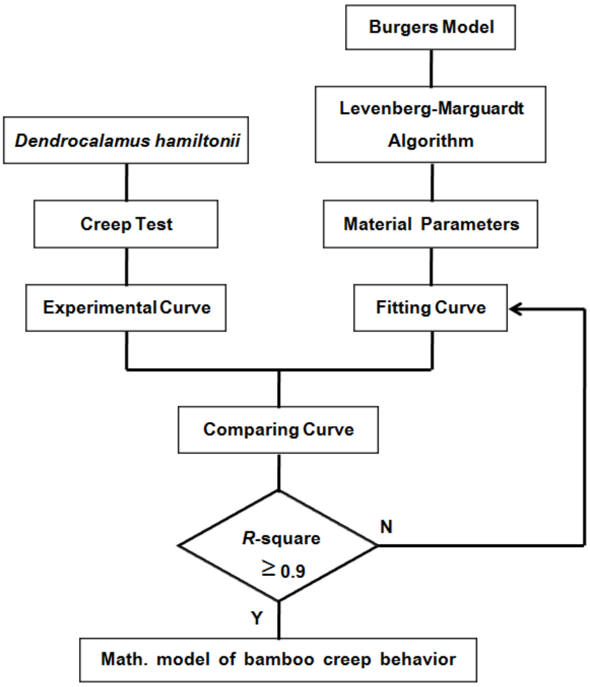

Fig. 6 Schematic of the research methodology.

Table. 1 Summary of the material model parameters with different stress levels for bamboo culm.

$\%S_{ut}$	Material model parameters									
	σ (MPa)	E_M (GPa)	E_{KV} (GPa)	η_{KV} (GPa·s)	η_M (GPa·s)	n	ε_{KV} (%)	η_r (s)	k	ε_P (%)
20%	17.298	232.032	3.629	5.587	892.109	0.046	0.030	0.648	0.423	0.012
30%	25.946	348.456	3.566	5.766	1347.844	0.041	0.148	0.532	0.120	0.013
40%	34.595	477.568	3.452	5.969	1802.762	0.040	0.157	0.329	0.114	0.046

3. Results and Discussion

In this paper, the bending creep data taken from the authors' previous work [11] were compared with the constitutive material model. It is noticed that the Burgers model was used in this study. Plots of the representative creep and creep-recovery strain vs. time at the various stress levels are shown in Fig. 7.

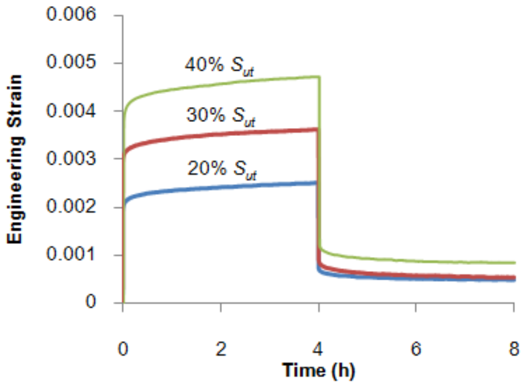

Fig. 7 Creep and creep-recovery tests data for bamboo culm at different stress levels [11].

Creep of the bamboo culm samples shows the typical viscoelastic behavior as followed: creep strain gradually increases with time and the applied stress, and the residual strains become noticeable under high loads even after a time period of active loading.

Using the Levenberg-Marquardt least squares method, the experimental data for each experiment were used to constructed Eq. (3) and (4) in order to obtain an appropriate value for each model parameters. The values of material model parameters for bamboo culm are summarized in Table. 1. The models of the creep strain and the creep-recovery strain are plotted and compared with the experimental results as shown Figs. 8 and 9, respectively.

It is noticed that the models can fitted the experimental results with high value of coefficient of determination (R-square) as shown.

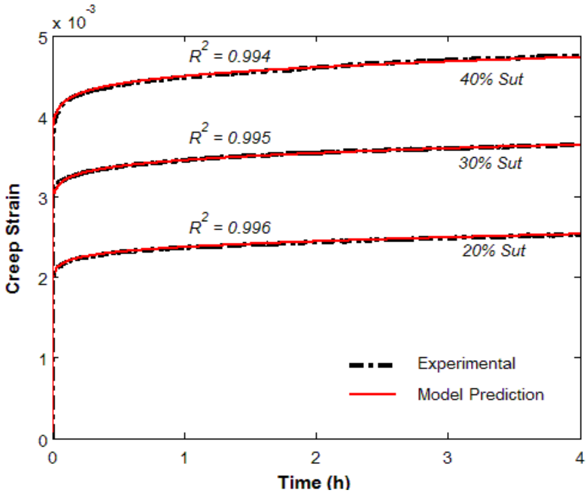

Fig. 8 Model prediction and experimental data for creep strain responses.

The experimental curves of creep phase were fitted by means of the Burgers model as shown in Fig. 8. The nonlinear curve fit function of the Levenberg-Marquardt algorithm was used with five parameters: E_M, η_M, E_{KV}, η_{KV}, and n. As shown in Fig.8, the model shows a satisfactory agreement with the experimental data. The parameters E_M, η_{KV} and η_M show an increasing trend with the applied stress. The parameter E_M associated to the Maxwell spring establishes instantaneous creep strain that would be recovered after stress elimination. The result shows that the applied stress has the ability to make the strength of the bamboo culm decreases, which is consistent with the results of bending creep tests. The parameter η_{KV} indicates that the viscosity of the Kelvin-Voigt unit and the ratio of η_{KV} to E_{KV} are the retardation time. The parameter η_{KV} increases with the applied stress. One more important parameter is η_M that is also listed in Table 1. It represents the irrecoverable creep strain. It is noticed that the value of the parameter is much higher than that of η_{KV}, and it is also sensitive to the applied stress. As seen from the fitting results, the viscosity η_M increases with applied stress and the permanent deformation. On the other hand, the retardant elasticity E_{KV} is related to the stiffness of the bamboo culm, and it also decreases with the applied stress.

The fits of the experimental curves of the creep-recovery strain as a function of time are shown in Fig. 9. On repeating the fits with these parameters, the strain values of the Maxwell dashpot (ε_{KV}) and of the Kelvin-Voigt elements (ε_P) can be calculated. The results, obtained together with the parameters η_r and k, are shown in Table 1. It is apparent that the values of ε_{KV} and ε_P increase with the applied stress. These parameters can reflect the change of the creep rate of bamboo culm when applied stress is changed. On the other hand, the values of η_r and k decrease with the applied stress that indicates a reduced recovery performance.

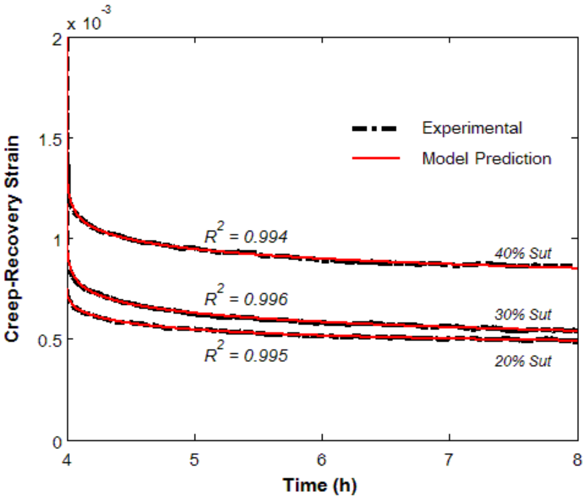

Fig. 9 Model prediction and experimental data for creep-recovery strain responses.

Combining the analysis of the two parts indicates that increasing applied stress may be lead to an increase in viscosity in the creep part, and it may be lead to a decrease in viscosity in the creep-recovery part. These behaviors are influenced by the parameters η_M and η_r, respectively. As a result, the permanent strain increases, which reflects by the parameter ε_P. It is noticed that the permanent creep strain ε_P is the important parameter which directly relates to the creep-recovery property of materials. Thus increasing the viscosity makes the permanent strain and creep-recovery strain rising.

A good agreement between experimental and predicted curves shows that the assumption of viscoelastic behavior used in this study is practicable. This can be considered by the values of *R*-squares. Therefore, it can be confirmed that the Levenberg-Marguardt algorithm is usable for solving the material model parameters of bamboo culm. The model used in this study are also shown good agreement with the models of other viscoelastic materials, such as, the analytically for the polypropylene/multi-walled carbon nanotube composites as did by Jia *et al*. [6], bamboo fiber high-density polyethylene (BF/HDPE) composite materials as did by Su *et al*. [8], and white spruce (*Piceaglauca* (Moench.) Voss.) wood as did by Moutee [9].

4. Conclusions

In this present work, the material model parameters identification of the bamboo culm behavior by the Burgers model was the main aim. A certain number of creep tests were conducted at different stress levels. Experimental results were used to construct the material model parameters identification. The material behavior was represented by Burgers model associated with Weibull distribution equation for describing the viscoelastic behavior. It can be shown in this study that the Levenberg-Marguardt algorithm is suitable for solving these material model parameters of the bamboo culm. This algorithm shows good capability in predicting nonlinear viscoelastic behavior under bending load condition. Both Burgers model associated with Weibull distribution and Levenberg-Marquardt algorithm were used to predict the creep behavior of the bamboo wood in the structural member of the local house and building.

5. Acknowledgement

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